6.3.4 Frictional Loss in Separated Flow

Having discussed homogeneous and disperse flows, attention will now be turned to the friction in separated flows and, in particular, describe the commonly used Martinelli correlations. The Lockhart-Martinelli (Lockhart and Martinelli, 1949) and Martinelli-Nelson (Martinelli and Nelson, 1948) correlations are widely documented in multiphase flow texts (see, for example, Wallis 1969 or Brennen 2005). These attempt to predict the frictional pressure gradient in two-component or two-phase pipe flows. It is assumed that these flows consist of two separate co-current streams that, for convenience, will be referred to as the liquid and the gas though they could be any two immiscible fluids. The correlations use the results for the frictional pressure gradient in single phase pipe flows of each of the two fluid streams. In two-phase flow, the volume fraction is often changing as the mixture progresses along the pipe and such phase change necessarily implies acceleration or deceleration of the fluids. Associated with this acceleration is an additional acceleration component of the pressure gradient that is addressed with the Martinelli-Nelson correlation. Obviously, it is convenient to begin with the simpler, two-component case (the Lockhart-Martinelli correlation); this also neglects the effects of changes in the fluid densities with distance, s, along the pipe axis so that the fluid velocities also remain invariant with s. Moreover, in all cases, it is assumed that the hydrostatic pressure gradient has been accounted for so that the only remaining contribution to the pressure gradient, $-dp/ds$, is that due to the wall shear stress, τ_w. A simple balance of forces requires that

$$-\frac{dp}{ds} = \frac{P}{A}\tau_w \tag{1}$$

where P and A are the perimeter and cross-sectional area of the stream or pipe. For a circular stream or pipe, $P/A = 4/d$, where d is the stream/pipe diameter. For non-circular cross-sections, it is convenient to define a *hydraulic diameter*, $4A/P$. Then, defining the dimensionless friction coefficient, C_f, as

$$C_f = \tau_w / \frac{1}{2}\rho j^2 \tag{2}$$

the more general form of equation 1, section 6.3.2, becomes

$$-\frac{dp}{ds} = C_f \rho j^2 \frac{P}{2A} \tag{3}$$

In single phase flow the coefficient, C_f, is a function of the Reynolds number, $\rho d j/\mu$, of the form

$$C_f = \mathcal{K}\left\{\frac{\rho d j}{\mu}\right\}^{-m} \tag{4}$$

where $\mathcal{K}$ is a constant that depends on the roughness of the pipe surface and will be different for laminar and turbulent flow. The index, m, is also different, being 1 in the case of laminar flow and 1/4 in the case of turbulent flow.

These relations from single phase flow are applied to the two cocurrent streams in the following way. First, hydraulic diameters, d_L and d_G, will be defined for each of the two streams and the corresponding area ratios, κ_L and κ_G, are then given by

$$\kappa_L = 4A_L/\pi d_L^2 \quad ; \quad \kappa_G = 4A_G/\pi d_G^2 \tag{5}$$

where $A_L = A(1-\alpha)$ and $A_G = A\alpha$ are the actual cross-sectional areas of the two streams. The quantities κ_L and κ_G are shape parameters that depend on the geometry of the flow pattern. In the absence of any specific information on this geometry, one might choose the values pertinent to streams of circular cross-section, namely $\kappa_L = \kappa_G = 1$, and the commonly used form of the Lockhart-Martinelli correlation employs these values. (Note that Brennen (2005) also presents results for an alternative choice.)

The basic geometric relations yield

$$\alpha = 1 - \kappa_L d_L^2/d^2 = \kappa_G d_G^2/d^2 \tag{6}$$

Then, the pressure gradient in each stream is assumed given by the following coefficients taken from single phase pipe flow:

$$C_{fL} = \mathcal{K}_L \left\{ \frac{\rho_L d_L u_L}{\mu_L} \right\}^{-m_L} \quad ; \quad C_{fG} = \mathcal{K}_G \left\{ \frac{\rho_G d_G u_G}{\mu_G} \right\}^{-m_G} \tag{7}$$

and, since the pressure gradients must be the same in the two streams, this imposes the following relation between the flows:

$$-\frac{dp}{ds} = \frac{2\rho_L u_L^2 \mathcal{K}_L}{d_L} \left\{ \frac{\rho_L d_L u_L}{\mu_L} \right\}^{-m_L} = \frac{2\rho_G u_G^2 \mathcal{K}_G}{d_G} \left\{ \frac{\rho_G d_G u_G}{\mu_G} \right\}^{-m_G} \tag{8}$$

In the above, m_L and m_G are 1 or 1/4 depending on whether the stream is laminar or turbulent.

Equations 6 and 8 are the basic relations used to construct the Lockhart-Martinelli correlation. The solutions to these equations are normally and most conveniently presented in non-dimensional form by defining the following dimensionless pressure gradient parameters:

$$\phi_L^2 = \frac{\left(\frac{dp}{ds}\right)_{actual}}{\left(\frac{dp}{ds}\right)_L} \quad ; \quad \phi_G^2 = \frac{\left(\frac{dp}{ds}\right)_{actual}}{\left(\frac{dp}{ds}\right)_G} \tag{9}$$

where $(dp/ds)_L$ and $(dp/ds)_G$ are respectively the hypothetical pressure gradients that would occur in the same pipe if only the liquid flow were present and if only the gas flow were present. The ratio of these two hypothetical gradients, Ma, given by

$$Ma^2 = \frac{\phi_G^2}{\phi_L^2} = \frac{\left(\frac{dp}{ds}\right)_L}{\left(\frac{dp}{ds}\right)_G} = \frac{\rho_G j_G^2}{\rho_L j_L^2} \frac{\mathcal{K}_G}{\mathcal{K}_L} \frac{\left\{\frac{\rho_G j_G d}{A\mu_G}\right\}^{-m_G}}{\left\{\frac{\rho_L j_L d}{A\mu_L}\right\}^{-m_L}} \tag{10}$$

has come to be called the Martinelli parameter and allows presentation of the solutions to equations 6 and 8 in a convenient parametric form. Using the definitions of equations 9, the non-dimensional forms of equations 6 become

$$\alpha = 1 - \kappa_L^{(3-m_L)/(m_L-5)} \phi_L^{4/(m_L-5)} = \kappa_G^{(3-m_G)/(m_G-5)} \phi_G^{4/(m_G-5)} \qquad (11)$$

and the solution of these equations produces the Lockhart-Martinelli prediction of the non-dimensional pressure gradient.

To summarize: for given values of (a) the fluid properties, ρ_L, ρ_G, μ_L and μ_G (b) the nature of the flow, laminar or turbulent, in the two streams and the phase correlation constants, m_L, m_G, $\mathcal{K}_L$ and $\mathcal{K}_G$ (c) the parameters defined by the flow pattern geometry, κ_L and κ_G and (d) a given value of α equations 11 can be solved to find the non-dimensional solution to the flow, namely the values of ϕ_L^2 and ϕ_G^2. The value of Ma^2 also follows and the rightmost expression in equation 10 then yields a relation between the liquid mass flux, $\rho_L j_L$, and the gas mass flux, $\rho_G j_G$. Thus, if one is also given just **one** mass flux (often this will be the total mass flux, $\dot{m} = \rho_L j_L + \rho_G j_G$), the solution will yield the individual mass fluxes, the mass quality and other flow properties. Alternatively one could begin the calculation with the mass quality rather than the void fraction and find the void fraction as one of the results. Finally the pressure gradient, dp/ds, follows from the values of ϕ_L^2 and ϕ_G^2.

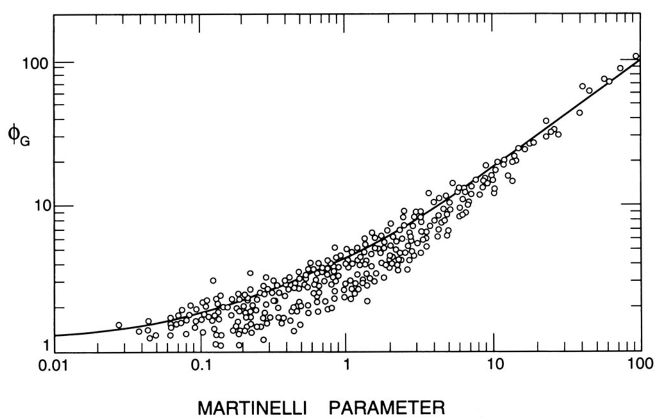

Figure 1: Comparison of the Lockhart-Martinelli correlation (the TT case) for ϕ_G (solid line) with experimental data. Adapted from Turner and Wallis (1965).

Charts for the results are presented by Wallis (1969), Brennen (2005) and others. Charts like these are commonly used in the manner described above to obtain solutions for two-component gas/liquid flows in pipes. A typical comparison of the Lockhart-Martinelli prediction with the experimental data is presented in figure 1. Note that the scatter in the data is significant (about a factor of 3 in ϕ_G) and that the Lockhart-Martinelli prediction often yields an overestimate of the friction or pressure gradient. This is the result of the assumption that the entire perimeter of both phases experiences static wall friction. This is not the case and part of the perimeter of each phase is in contact with the other phase. If the interface is smooth this could result in a decrease in the friction; on the other hand a roughened interface could also result in increased interfacial friction.

It is important to recognize that there are many deficiencies in the Lockhart-Martinelli approach. First, it is assumed that the flow pattern consists of two parallel streams and any departure from this topology could result in substantial errors. Second, there is the previously discussed deficiency regarding the suitability of assuming that the perimeters of both phases experience friction that is effectively equivalent to that of a static solid wall. A third source of error arises because the multiphase flows are often unsteady and this yields a multitude of quadratic interaction terms that contribute to the mean flow in the same way that Reynolds stress terms contribute to turbulent single phase flow.

The Lockhart-Martinelli correlation was extended by Martinelli and Nelson (1948) to include the effects of phase change. This extension includes evaluation of the additional pressure gradient due to the acceleration of the flow caused by the phase change. To evaluate this one must know the variation of the mass quality, $\mathcal{X}$, with distance, s, along the pipe. In many boilers, evaporators or condensers, the rate of heat supply or removal per unit length of the pipe, $\mathcal{Q}_\ell$, is roughly uniform and the latent heat, $\mathcal{L}$, can be also be considered constant. It follows that for a flow rate of $\dot{m}$ in a pipe of cross-sectional area, A, the mass quality varies linearly with distance, s, since

$$\frac{d\mathcal{X}}{ds} = \frac{\mathcal{Q}_\ell}{A\dot{m}\mathcal{L}} \tag{12}$$

Given the quantities on the right-hand side this allows evaluation of the mass quality as a function of distance along the conduit and also allows evaluation of the additional acceleration contributions to the pressure gradient. For further details the reader is referred to Brennen (2005).