Non-Cavitating Pumps

Consider now the questions associated with transfer functions for pumps or other turbomachines. In the simple fluid flows of the section "Some Simple Transfer Matrices" we were able to utilize the known equations governing the flow in order to construct the transfer functions for those simple components. In the case of more complex fluids or geometries, one cannot necessarily construct appropriate one-dimensional flow equations, and therefore must resort to results derived from more global application of conservation laws or to experimental measurements of transfer matrices. Consider first the transfer matrix, $[TP]$, for incompressible flow through a pump (all pump transfer functions will be of the $[T]$ form defined in equation (Bngc6)) which will clearly be a function not only of the frequency, ω, but also of the mean operating point as represented by the flow coefficient, ϕ, and the cavitation number, σ. At very low frequencies one can argue that the pump will simply track up and down the performance characteristic, so that, for small amplitude perturbations and in the absence of cavitation, the transfer function becomes

$$[TP] = \begin{bmatrix} 1 & \frac{d(\Delta p^T)}{dm} \\ 0 & 1 \end{bmatrix} \tag{Nrp1}$$

where $d(\Delta p^T)/dm$ is the slope of the steady state operating characteristic of total pressure rise versus mass flow rate. Thus we define the pump resistance, $R_P = -d(\Delta p^T)/dm$, where R_P is usually positive under design flow conditions, but may be negative at low flow rates as discussed earlier (see the section on "Surge"). At finite frequencies, the elements TP_{21} and TP_{22} will continue to be zero and unity respectively, since the instantaneous flow rate into and out of the pump must be identical when the fluid and structure are incompressible and no cavitation occurs. Furthermore, TP_{11} must continue to be unity since, in an incompressible flow, the total pressure differences must be independent of the level of the pressure. It follows that the transfer function at higher frequencies will become

$$[TP] = \begin{bmatrix} 1 & -I_P \\ 0 & 1 \end{bmatrix} \tag{Nrp2}$$

where the pump impedence, I_P, will, in general, consist of a resistive part, R_P, and a reactive part, $j\omega L_P$. The resistance, R_P, and inertance, L_P, could be functions of both the frequency, ω, and the mean flow conditions. Such simple impedance models for pumps have been employed, together with transfer functions for the suction and discharge lines (equation (Bngg12)), to model the dynamics of pumping systems. For example, Dussourd (1968) used frequency domain methods to analyse pulsation problems in boiler feed pump systems. More recently, Sano (1983) used transfer functions to obtain natural frequencies for pumping systems that agree well with those observed experimentally.

The first fundamental investigation of the dynamic response of pumps seems to have been carried out by Ohashi (1968) who analyzed the oscillating flow through a cascade, and carried out some preliminary experimental investigations on a centrifugal pump. These studies enabled him to evaluate the frequency at which the response of the pump would cease to be quasistatic (see below). Fanelli (1972) appears to have been the first to explore the nature of the pump transfer function, while the first systematic measurements of the impedance of a noncavitating centrifugal pump are those of Anderson, Blade and Stevans (1971). Typical resistive and reactive component measurements from the work of Anderson, Blade and Stevans are reproduced in figure 1. Note that, though the resistance approaches the quasistatic value at low frequencies, it also departs significantly from this value at higher frequencies. Moreover, the reactive part is only roughly linear with frequency. The resistance and inertance are presented again in figure 2, where

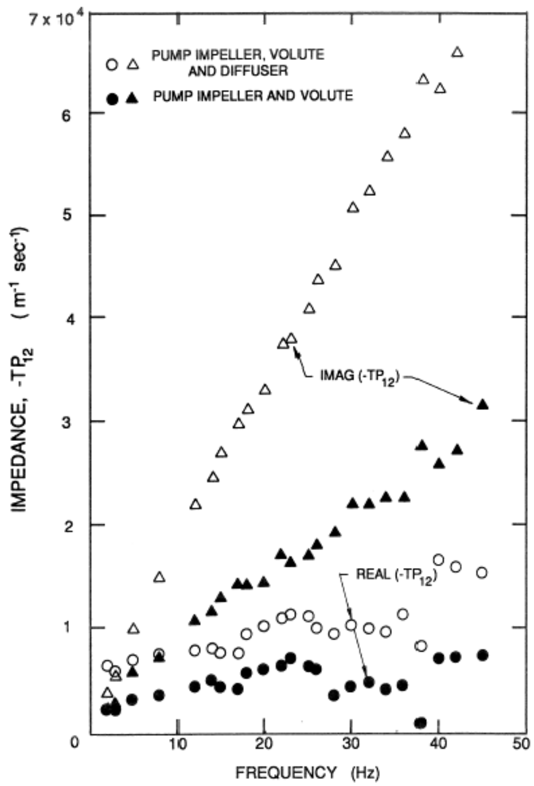

Figure 1: Impedance measurements made by Anderson, Blade and Stevans (1971) on a centrifugal pump (impeller diameter of 18.9 cm) operating at a flow coefficient of 0.442 and a speed of 3000 rpm. The real or resistive part of $(-T_{12})$ and the imaginary or reactive part of (T_{12}) are plotted against the frequency of the perturbation.

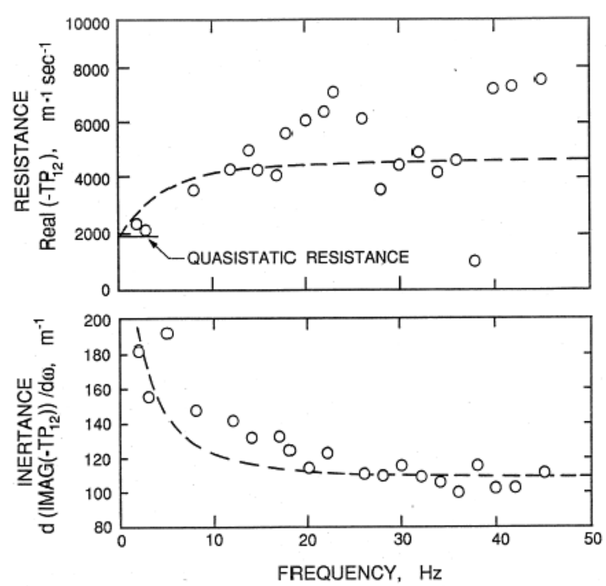

Figure 2: Typical inertance and resistance values from the centrifugal pump data of figure 1. Data do not include the diffuser contribution. The lines correspond to analytical values obtained as described in the text.

they are compared with the results of a dynamic model proposed by Anderson, Blade and Stevans. In this model, each pump impeller passage is represented by a resistance and an inertance, and the volute by a series of resistances and inertances. Since each impeller passage discharges into the volute at different locations relative to the volute discharge, each impeller passage flow experiences a different impedance on its way to the discharge. This results in an overall pump resistance and inertance that are frequency dependent as shown in figure 2. Note that the comparison with the experimental observations (which are also included in figure 2) is fair, but not completely satisfactory. Moreover, it should be noted, that the comparison shown is for a flow coefficient of 0.442 (above the design flow coefficient), and that, at higher flow coefficients, the model and experimental results exhibited poorer agreement.

Subsequent measurements of the impedance of non-cavitating axial and mixed flow pumps by Ng and Brennen (1978) exhibit a similar increase in the resistance with frequency (see next section). In both sets of dynamic data, it does appear that significant departure from the quasistatic values can be expected when the reduced frequency, (frequency/rotation frequency) exceeds about 0.02 (see figure 2). This is roughly consistent with the criterion suggested by Ohashi (1968) who concluded that non-quasistatic effects would occur above a reduced frequency of $0.05 Z_R \phi / \cos\beta$. For the inducers of Ng and Brennen, Ohashi's criterion yields values for the critical reduced frequency of about 0.015.