

Single Droplet Combustion

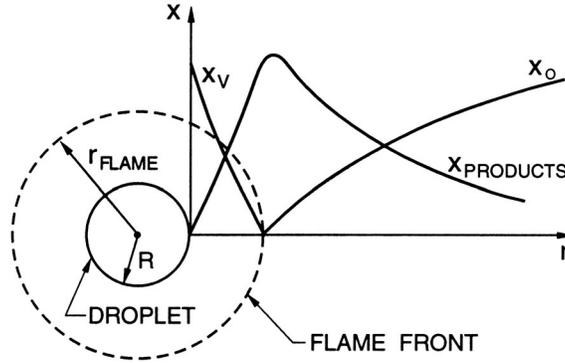


Figure 1: Schematic of single droplet combustion indicating the radial distributions of fuel/vapor mass fraction, x_V , oxidant mass fraction, x_O , and combustion products mass fraction.

For very small droplets of a volatile fuel, droplet evaporation is completed early in the heating process and the subsequent combustion process is unchanged by the fact that the fuel began in droplet form. On the other hand for larger droplets or less volatile fuels, droplet evaporation will be a controlling process during combustion. Consequently, analysis of the combustion of a single droplet begins with the single droplet evaporation discussed in the preceding section. Then single droplet combustion consists of the outward diffusion of fuel vapor from the droplet surface and the inward diffusion of oxygen (or other oxidant) from the far field, with the two reacting in a flame front at a certain radius from the droplet. It is usually adequate to assume that this combustion occurs instantaneously in a thin *flame front* at a specific radius, r_{flame} , as indicated in figure 1. As in the last section, a steady state process will be assumed in which the mass rates of consumption of fuel and oxidant in the flame are denoted by \dot{m}_{VC} and \dot{m}_{OC} respectively. For combustion stoichiometry we therefore have

$$\dot{m}_{VC} = \nu \dot{m}_{OC} \quad (\text{Noh1})$$

where ν is the mass-based stoichiometric coefficient for complete combustion. Moreover the rate of heat release due to combustion will be $\mathcal{Q}\dot{m}_{VC}$ where \mathcal{Q} is the combustion heat release per unit mass of fuel. Assuming the mass diffusivities for the fuel and oxidant and the thermal diffusivity ($k/\rho c_p$) are all the same (a Lewis number of unity) and denoted by D , the thermal and mass conservation equations for this process can then be written as:

$$\dot{m}_V \frac{dT}{dr} = \frac{d}{dr} \left(4\pi r^2 \rho D \frac{dT}{dr} \right) + 4\pi r^2 \frac{\mathcal{Q}\dot{m}_{VC}}{c_p} \quad (\text{Noh2})$$

$$\dot{m}_V \frac{dx_V}{dr} = \frac{d}{dr} \left(4\pi r^2 \rho D \frac{dx_V}{dr} \right) + 4\pi r^2 \dot{m}_{VC} \quad (\text{Noh3})$$

$$\dot{m}_V \frac{dx_O}{dr} = \frac{d}{dr} \left(4\pi r^2 \rho D \frac{dx_O}{dr} \right) - 4\pi r^2 \dot{m}_{OC} \quad (\text{Noh4})$$

where x_O is the mass fraction of oxidant.

Using equation (Noh1) to eliminate the reaction rate terms these become

$$\dot{m}_V \frac{d}{dr}(c_p T + \mathcal{Q} x_V) = \frac{d}{dr} \left(4\pi r^2 \rho D \frac{d}{dr}(c_p T + \mathcal{Q} x_V) \right) \quad (\text{Noh5})$$

$$\dot{m}_V \frac{d}{dr}(c_p T + \nu \mathcal{Q} x_O) = \frac{d}{dr} \left(4\pi r^2 \rho D \frac{d}{dr}(c_p T + \nu \mathcal{Q} x_O) \right) \quad (\text{Noh6})$$

$$\dot{m}_V \frac{d}{dr}(x_V - \nu x_O) = \frac{d}{dr} \left(4\pi r^2 \rho D \frac{d}{dr}(x_V - \nu x_O) \right) \quad (\text{Noh7})$$

Appropriate boundary conditions on these relations are (1) the droplet surface heat flux condition (Nog6), (2) zero droplet surface flux of non-fuel gases from equations (Nog1) and (Nog2), (3) zero oxidant flux at the droplet surface, (4) zero oxidant mass fraction at the droplet surface (5) temperature at the droplet surface, $T_{r=R}$, (6) known temperature far from the flame, $T_{r=\infty}$, (7) zero fuel/vapor mass fraction far from the flame, $(x_V)_{r=\infty} = 0$, and (8) a known oxidant mass fraction far from the flame, $(x_O)_{r=\infty}$. Using these conditions equations (Noh5), (Noh6) and (Noh7) may be integrated twice to obtain:

$$\frac{\dot{m}_V}{4\pi\rho D r} = \ln \left\{ \frac{c_p(T_{r=\infty} - T_{r=R}) + \mathcal{L} - \mathcal{Q}}{c_p(T - T_{r=R}) + \mathcal{L} - \mathcal{Q}(1 - x_V)} \right\} \quad (\text{Noh8})$$

$$\frac{\dot{m}_V}{4\pi\rho D r} = \ln \left\{ \frac{c_p(T_{r=\infty} - T_{r=R}) + \mathcal{L} + \nu \mathcal{Q}(x_O)_{r=\infty}}{c_p(T - T_{r=R}) + \mathcal{L} + \nu \mathcal{Q} x_O} \right\} \quad (\text{Noh9})$$

$$\frac{\dot{m}_V}{4\pi\rho D r} = \ln \left\{ \frac{1 + \nu(x_O)_{r=\infty}}{1 - x_V + \nu x_O} \right\} \quad (\text{Noh10})$$

and evaluating these expressions at the droplet surface leads to:

$$\frac{\dot{m}_V}{4\pi\rho D R} = \ln \left\{ \frac{c_p(T_{r=\infty} - T_{r=R}) + \mathcal{L} - \mathcal{Q}}{\mathcal{L} - \mathcal{Q}(1 - (x_V)_{r=R})} \right\} \quad (\text{Noh11})$$

$$\frac{\dot{m}_V}{4\pi\rho D R} = \ln \left\{ \frac{c_p(T_{r=\infty} - T_{r=R}) + \mathcal{L} + \nu \mathcal{Q}(x_O)_{r=\infty}}{\mathcal{L}} \right\} \quad (\text{Noh12})$$

$$\frac{\dot{m}_V}{4\pi\rho D R} = \ln \left\{ \frac{1 + \nu(x_O)_{r=\infty}}{1 - (x_V)_{r=R}} \right\} \quad (\text{Noh13})$$

and consequently the unknown surface conditions, $T_{r=R}$ and $(x_V)_{r=R}$ may be obtained from the relations

$$\begin{aligned} \frac{1 + \nu(x_O)_{r=\infty}}{1 - (x_V)_{r=R}} &= \frac{c_p(T_{r=\infty} - T_{r=R}) + \mathcal{L} + \nu \mathcal{Q}(x_O)_{r=\infty}}{\mathcal{L}} \\ &= \frac{c_p(T_{r=\infty} - T_{r=R}) + \mathcal{L} - \mathcal{Q}}{\mathcal{L} - \mathcal{Q}(1 - (x_V)_{r=R})} \end{aligned} \quad (\text{Noh14})$$

Having solved for these surface conditions, the evaporation rate, \dot{m}_V , would follow from any one of equations (Noh11) to (Noh13). However a simple, approximate expression for \dot{m}_V follows from equation (Noh12) since the term $c_p(T_{r=\infty} - T_{r=R})$ is generally small compared with $\mathcal{Q}(x_V)_{r=R}$. Then

$$\dot{m}_V \approx 4\pi R \rho D \ln \left(1 + \frac{\nu \mathcal{Q}(x_O)_{r=\infty}}{\mathcal{L}} \right) \quad (\text{Noh15})$$

The position of the flame front, $r = r_{flame}$, follows from equation (Noh13) by setting $x_V = x_O = 0$:

$$r_{flame} = \frac{\dot{m}_V}{4\pi\rho D \ln(1 + \nu(x_O)_{r=\infty})} \approx R \frac{\ln(1 + \nu \mathcal{Q}(x_O)_{r=\infty}/\mathcal{L})}{\ln(1 + \nu(x_O)_{r=\infty})} \quad (\text{Noh16})$$

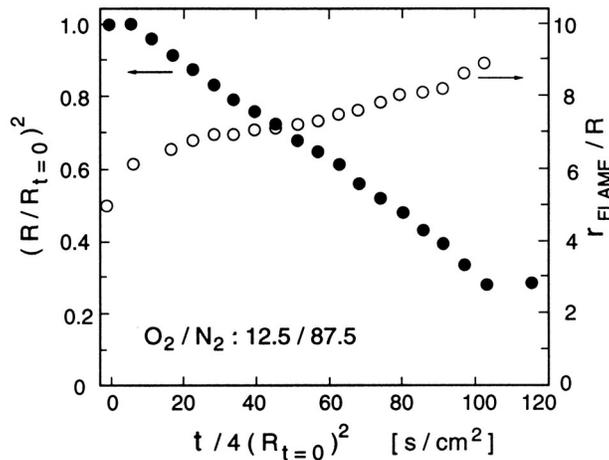


Figure 2: Droplet radius, R , and the ratio of the flame radius to the droplet radius, r_{flame}/R , for a burning octane droplet in a 12.5% O_2 , 87.5% N_2 , 0.15atm environment. Adapted from Law (1982).

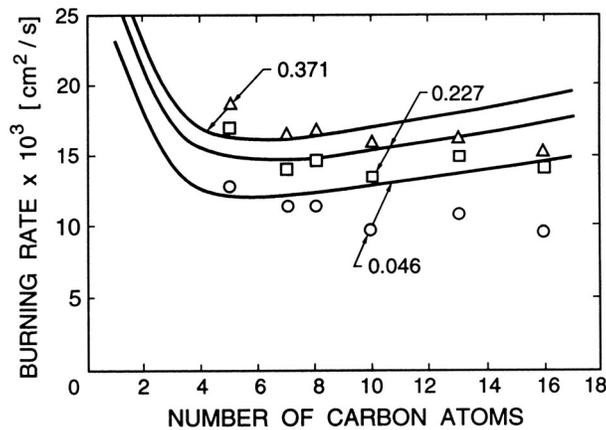


Figure 3: Theoretical and experimental burning rates, $-d(R^2)/dt$ (in cm^2/s), of various paraffin hydrocarbon droplets ($R = 550\mu m$) in a $T_{r=\infty} = 2530^\circ K$ environment with various mass fractions of oxygen, $(x_O)_{r=\infty}$, as shown. Adapted from Faeth and Lazar (1971).

As one might expect, the radius of the flame front increases rapidly at small oxygen concentrations, $(x_O)_\infty$, since this oxygen is quickly consumed. However, the second expression demonstrates that r_{flame}/R is primarily a function of Q/\mathcal{L} ; indeed for small values of $(x_O)_{r=\infty}$ it follows that $r_{flame}/R \approx Q/\mathcal{L}$. We discuss the consequences of this in the next section.

Detailed reviews of the corresponding experimental data on single droplet combustion can be found in numerous texts and review articles including those listed above. Here we include just two sets of experimental results. Figure 2 exemplifies the data on the time history of the droplet radius, R , and the ratio of the flame radius to the droplet radius, r_{flame}/R . Note that after a small initial transient, R^2 decreases quite linearly with time as explicitly predicted by equation (Nog10) and implicitly contained in the combustion analysis. The slope, $-d(R^2)/dt$, is termed the *burning rate* and examples of the comparison between the theoretical and experimental burning rates are included in figure 3. The flame front location is also shown in figure 2; note that r_{flame}/R is reasonably constant despite the fivefold shrinkage of the droplet.

Further refinements of this simple analysis can also be found in the texts mentioned previously. A few of the assumptions that require further analysis include whether or not the assumed steady state is pertinent, whether relative motion of the droplets through the gas convectively enhances the heat and mass transfer

processes, the role of turbulence in modifying the heat and mass transfer processes in the gas, whether the chemistry can be modeled by a simple flame front, the complexity introduced by mixtures of liquids of different volatilities, and whether all the diffusivities can be assumed to be similar.