Flow around a Sphere at High Reynolds Number

For steady flows about a sphere in which $dU_i/dt = dV_i/dt = dW_i/dt = 0$, it is convenient to use a coordinate system, x_i, fixed in the particle as well as polar coordinates (r, θ) and velocities u_r, u_θ as defined in figure 1.

Then equations (Nea1) and (Nea2) become

$$\frac{1}{r^2}\ \frac{\partial}{\partial r}(r^2 u_r) + \frac{1}{r\sin\theta}\ \frac{\partial}{\partial\theta}(u_\theta \sin\theta) = 0 \tag{Neb1}$$

and

$$\rho_C \left\{ \frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} \right\} = -\frac{\partial p}{\partial r}$$
$$+\rho_C \nu_C \left\{ \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial u_r}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial u_r}{\partial\theta}\right) - \frac{2u_r}{r^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial\theta} \right\} \tag{Neb2}$$

$$\rho_C \left\{ \frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} \right\} = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$
$$+\rho_C \nu_C \left\{ \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial u_\theta}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial u_\theta}{\partial\theta}\right) + \frac{2}{r^2}\frac{\partial u_r}{\partial\theta} - \frac{u_\theta}{r^2\sin^2\theta} \right\} \tag{Neb3}$$

The Stokes streamfunction, ψ, is defined to satisfy continuity automatically:

$$u_r = \frac{1}{r^2 \sin\theta}\frac{\partial\psi}{\partial\theta} \quad ; \quad u_\theta = -\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r} \tag{Neb4}$$

and the inviscid potential flow solution is

$$\psi = -\frac{W r^2}{2}\sin^2\theta - \frac{D}{r}\sin^2\theta \tag{Neb5}$$

$$u_r = -W\cos\theta - \frac{2D}{r^3}\cos\theta \tag{Neb6}$$

$$u_\theta = +W\sin\theta - \frac{D}{r^3}\sin\theta \tag{Neb7}$$

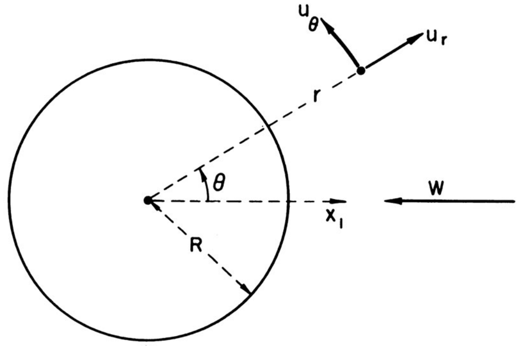

Figure 1: Notation for a spherical particle.

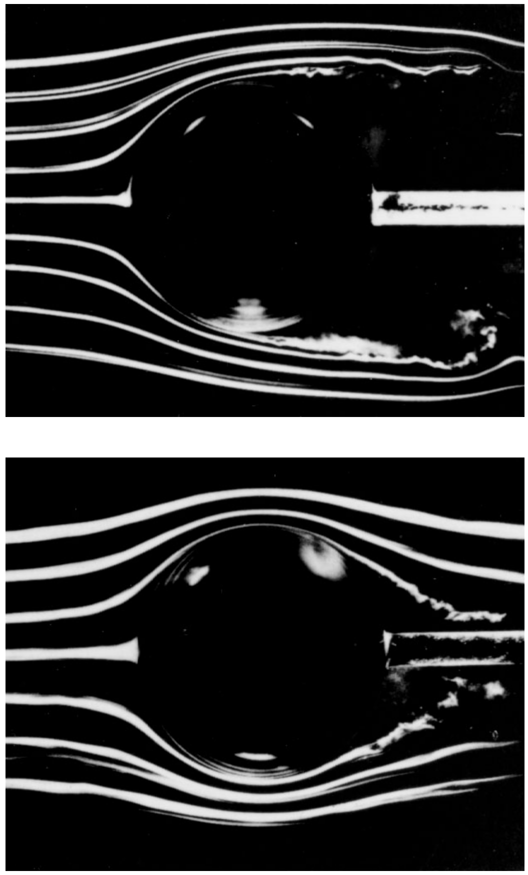

Figure 2: Smoke visualization of the nominally steady flows (from left to right) past a sphere showing, at the top, laminar separation at $Re = 2.8 \times 10^5$ and, on the bottom, turbulent separation at $Re = 3.9 \times 10^5$. Photographs by F.N.M.Brown, reproduced with the permission of the University of Notre Dame.

$$\phi = -Wr\cos\theta + \frac{D}{r^2}\cos\theta \qquad \text{(Neb8)}$$

where, because of the boundary condition $(u_r)_{r=R} = 0$, it follows that $D = -WR^3/2$. In potential flow one may also define a velocity potential, ϕ, such that $u_i = \partial\phi/\partial x_i$. The classic problem with such solutions is the fact that the drag is zero, a circumstance termed D'Alembert's paradox. The flow is symmetric about the x_2x_3 plane through the origin and there is no wake.

The real viscous flows around a sphere at large Reynolds numbers, $Re = 2WR/\nu_C > 1$, are well documented. In the range from about 10^3 to 3×10^5, laminar boundary layer separation occurs at $\theta \cong 84°$ and a large wake is formed behind the sphere (see figure 2). Close to the sphere the *near-wake* is laminar; further downstream transition and turbulence occurring in the shear layers spreads to generate a turbulent *far-wake.* As the Reynolds number increases the shear layer transition moves forward until, quite abruptly, the turbulent shear layer reattaches to the body, resulting in a major change in the final position of separation ($\theta \cong 120°$) and in the form of the turbulent wake (figure 2). Associated with this change in flow pattern is a dramatic decrease in the drag coefficient, C_D (defined as the drag force on the body in the negative x_1 direction divided by $\frac{1}{2}\rho_C W^2 \pi R^2$), from a value of about 0.5 in the laminar separation regime to a value of about 0.2 in the turbulent separation regime (figure 3). At values of Re less than about 10^3 the flow becomes quite unsteady with periodic shedding of vortices from the sphere.

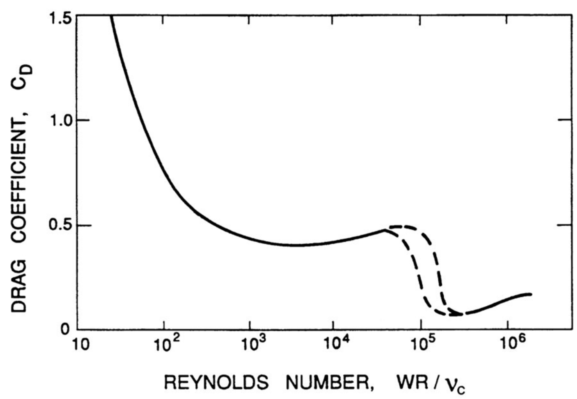

Figure 3: Drag coefficient on a sphere as a function of Reynolds number. Dashed curves indicate the drag crisis regime in which the drag is very sensitive to other factors such as the free stream turbulence.