Compressibility and Phase Change Effects

In this section the effects of the small pressure difference that must exist across a kinematic shock and the consequent effects of the corresponding density differences will be explored. The effects of phase change will also be explored.

By applying the momentum theorem to a control volume enclosing a portion of a kinematic shock in a frame of reference fixed in the shock, the following expression for the difference in the pressure across the shock is readily obtained:

$$p_2 - p_1 = \rho_A \left\{ \frac{(j'_{A1})^2}{\alpha_1} - \frac{(j'_{A2})^2}{\alpha_2} \right\} + \rho_B \left\{ \frac{(j'_{B1})^2}{(1-\alpha_1)} - \frac{(j'_{B2})^2}{(1-\alpha_2)} \right\} \tag{Nsg1}$$

Here we have assumed that any density differences that might occur will be second order effects. Since $j'_{A1} = j'_{A2}$ and $j'_{B1} = j'_{B2}$, it follows that

$$p_2 - p_1 = \frac{\rho_A (1-\alpha_1)(1-\alpha_2)}{(\alpha_1 - \alpha_2)} \left\{ \frac{j_{AB1}}{(1-\alpha_1)} - \frac{j_{AB2}}{(1-\alpha_2)} \right\}^2$$

$$- \frac{\rho_B \alpha_1 \alpha_2}{(\alpha_1 - \alpha_2)} \left\{ \frac{j_{AB1}}{\alpha_1} - \frac{j_{AB2}}{\alpha_2} \right\}^2 \tag{Nsg2}$$

Since the expressions inside the curly brackets are of order $(\alpha_1 - \alpha_2)$, the order of magnitude of $p_2 - p_1$ is given by

$$p_2 - p_1 = O\left((\alpha_1 - \alpha_2) \rho u_{AB}^2 \right) \tag{Nsg3}$$

provided neither α_1 nor α_2 are close to zero or unity. Here ρ is some representative density, for example the mixture density or the density of the heavier component. Therefore, provided the relative velocity, u_{AB}, is modest, the pressure difference across the kinematic shock is small. Consequently, the dynamic effects on the shock are small. If, under unusual circumstances, $(\alpha_1 - \alpha_2)\rho u_{AB}^2$ were to become significant compared with p_1 or p_2, the character of the shock would begin to change substantially.

Consider, now, the effects of the differences in density that the pressure difference given by equation (Nsg3) imply. Suppose that the component B is incompressible but that the component A is a compressible gas that behaves isothermally so that

$$\frac{\rho_{A1}}{\rho_{A2}} - 1 = \delta = \frac{p_1}{p_2} - 1 \tag{Nsg4}$$

If the kinematic shock analysis of section (Nse) is revised to incorporate a small density change ($\delta \ll 1$) in component A, the result is the following modification to equation (Nse4) for the shock speed:

$$u_s = j_1 + \frac{j_{AB2} - j_{AB1}}{\alpha_2 - \alpha_1} - \frac{\delta (1-\alpha_2)(j_{AB1}\alpha_2 - j_{AB2}\alpha_1)}{(\alpha_2 - \alpha_1)(\alpha_2 - \alpha_1 - \alpha_2(1-\alpha_1)\delta)} \tag{Nsg5}$$

where terms of order δ^2 have been neglected. The last term in equation (Nsg5) represents the first order modification to the propagation speed caused by the compressibility of component A. From equations (Nsg3) and (Nsg4), it follows that the order of magnitude of δ is $(\alpha_1 - \alpha_2)\rho u_{AB}^2/p$ where p is a representative pressure. Therefore, from equation (Nsg5), the order of magnitude of the correction to u_s is $\rho u_{AB}^3/p$ which is the typical velocity, u_{AB}, multiplied by a Mach number. Clearly, this is usually a negligible correction.

Another issue that may arise concerns the effect of phase change in the shock. A different modification to the kinematic shock analysis allows some evaluation of this effect. Assume that, within the shock, mass is transfered from the more dense component B (the liquid phase) to the component A (or vapor phase) at a condensation rate equal to I per unit area of the shock. Then, neglecting density differences, the kinematic shock analysis leads to the following modified form of equation (Nse4):

$$u_s = j_1 + \frac{j_{AB2} - j_{AB1}}{\alpha_2 - \alpha_1} + \frac{I}{(\alpha_1 - \alpha_2)} \left\{ \frac{(1 - \alpha_2)}{\rho_A} + \frac{\alpha_2}{\rho_B} \right\} \tag{Nsg6}$$

Since $\alpha_1 > \alpha_2$ it follows that the propagation speed increases as the condensation rate increases. Under these circumstances, it is clear that the propagation speed will become greater than j_{A1} or j_1 and that the flux of vapor (component A) will be down through the shock. Thus, we can visualize that the shock will evolve from a primarily kinematic shock to a much more rapidly propagating condensation shock (see section (Nlh)).