Vapor/Liquid Nozzle Flow

A barotropic relation, equation (Nle6), was constructed in section (Nle) for the case of two-phase flow and, in particular, for vapor/liquid flow. This may be used to synthesize nozzle flows in a manner similar to the two-component analysis of the last section. Since the approximation $\rho \approx \rho_L(1-\alpha)$ was used in deriving both equation (Nle6) and equation (Nlf10), we may eliminate $\alpha/(1-\alpha)$ from these equations to obtain the velocity, u, in terms of p/p_o:

$$\frac{\rho_L}{p_o}\frac{u^2}{2} = 1 - \frac{p}{p_o} + \frac{1}{(1-k_V)}\left[\frac{\alpha_o}{(1-\alpha_o)} + \frac{k_L p_o^{-\eta}}{(k_V-\eta)}\right]\left[1 - \left(\frac{p}{p_o}\right)^{1-k_V}\right]$$

$$-\frac{1}{(1-\eta)}\left[\frac{k_L p_o^{-\eta}}{(k_V-\eta)}\right]\left[1 - \left(\frac{p}{p_o}\right)^{1-\eta}\right] \qquad \text{(Nlg1)}$$

To find the relation for the critical pressure ratio, p_*/p_o, the velocity, u, must equated with the sonic velocity, c, as given by equation (Nle7):

$$\frac{c^2}{2} = \frac{p}{\rho_L}\frac{\left[1 + \left\{\frac{\alpha_o}{1-\alpha_o} + k_L\frac{p_o^{-\eta}}{(k_V-\eta)}\right\}\left(\frac{p_o}{p}\right)^{k_V} - \left\{k_L\frac{p_o^{-\eta}}{(k_V-\eta)}\right\}\left(\frac{p_o}{p}\right)^{\eta}\right]^2}{2\left[k_V\left\{\frac{\alpha_o}{(1-\alpha_o)} + \frac{k_L p_o^{-\eta}}{(k_V-\eta)}\right\}\left(\frac{p_o}{p}\right)^{k_V} - \eta\left\{\frac{k_L p_o^{-\eta}}{(k_V-\eta)}\right\}\left(\frac{p_o}{p}\right)^{\eta}\right]} \qquad \text{(Nlg2)}$$

Though algebraically complicated, the equation that results when the right-hand sides of equations (Nlg1) and (Nlg2) are equated can readily be solved numerically to obtain the critical pressure ratio, p_*/p_o, for a given fluid and given values of α_o, the reservoir pressure and the interacting fluid fractions ϵ_L and ϵ_V (see

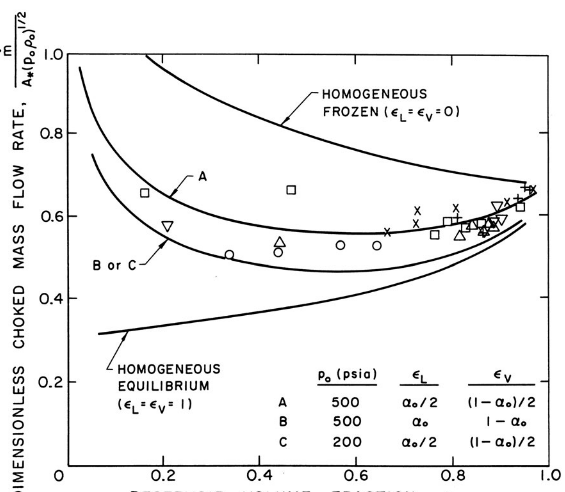

Figure 1: The dimensionless choked mass flow rate, $\dot{m}/A_*(p_o\rho_o)^{\frac{1}{2}}$, plotted against the reservoir vapor volume fraction, α_o, for water/steam mixtures. The data shown is from the experiments of Maneely (1962) and Neusen (1962) for $100 \rightarrow 200$ *psia* (+), $200 \rightarrow 300$ *psia* (×), $300 \rightarrow 400$ *psia* (⊡), $400 \rightarrow 500$ *psia* (△), $500 \rightarrow 600$ *psia* (▽) and > 600 *psia* (∗). The theoretical lines use $g^* = 1.67$, $\eta = 0.73$, $g_V = 0.91$, and $f_V = 0.769$ for water.

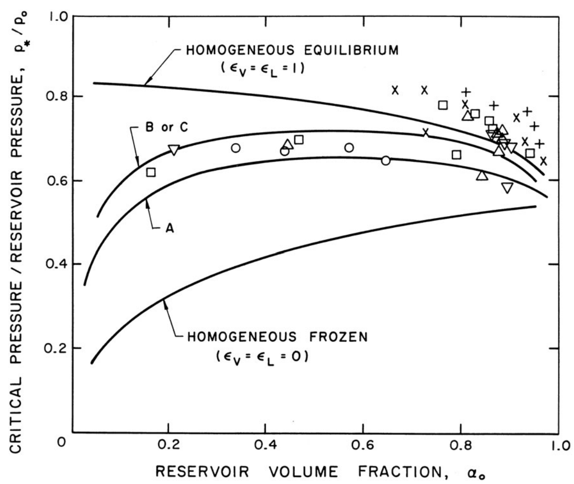

Figure 2: The ratio of critical pressure, p_*, to reservoir pressure, p_o, plotted against the reservoir vapor volume fraction, α_o, for water/steam mixtures. The data and the partially frozen model results are for the same conditions as in figure 1.

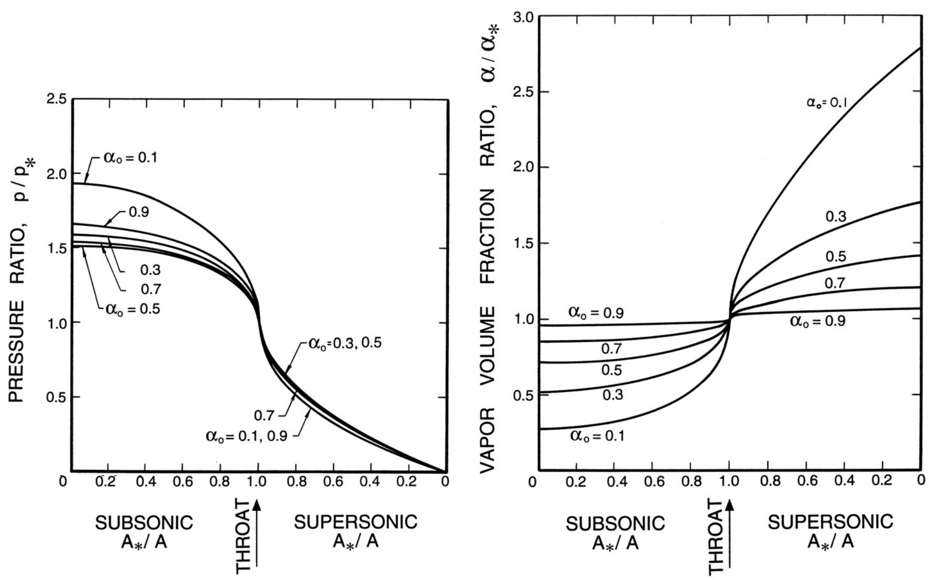

Figure 3: Left: Ratio of the pressure, p, to the critical pressure, p_*, and Right: Ratio of the vapor volume fraction, α, to the critical vapor volume fraction, α_*, as functions of the area ratio, A_*/A, for the case of water with $g^* = 1.67$, $\eta = 0.73$, $g_V = 0.91$, and $f_V = 0.769$.

section (Nld)). Having obtained the critical pressure ratio, the critical vapor volume fraction, α_*, follows from equation (Nle6) and the throat velocity, c_*, from equation (Nlg2). Then the dimensionless choked mass flow rate follows from the same relation as given in equation (Nlf12).

Sample results for the choked mass flow rate and the critical pressure ratio are shown in figures 1 and 2. Results for both homogeneous frozen flow ($\epsilon_L = \epsilon_V = 0$) and for homogeneous equilibrium flow ($\epsilon_L = \epsilon_V = 1$) are presented; note that these results are independent of the fluid or the reservoir pressure, p_o. Also shown in the figures are the theoretical results for various partially frozen cases for water at two different reservoir pressures. The interacting fluid fractions were chosen with the comment at the end of section (Nld) in mind. Since ϵ_L is most important at low vapor volume fractions (i.e., for bubbly flows), it is reasonable to estimate that the interacting volume of liquid surrounding each bubble will be of the same order as the bubble volume. Hence $\epsilon_L = \alpha_o$ or $\alpha_o/2$ are appropriate choices. Similarly, ϵ_V is most

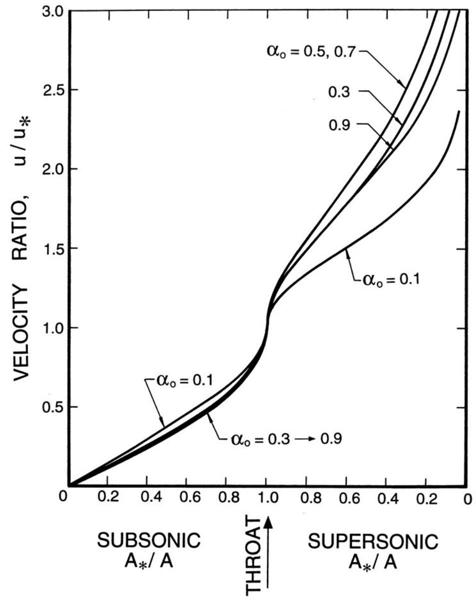

Figure 4: Ratio of the velocity, u, to the critical velocity, u_*, as a function of the area ratio for the same case as figure 3.

important at high vapor volume fractions (i.e., droplet flows), and it is reasonable to estimate that the interacting volume of vapor surrounding each droplet would be of the same order as the droplet volume; hence $\epsilon_V = (1 - \alpha_o)$ or $(1 - \alpha_o)/2$ are appropriate choices.

Figures 1 and 2 also include data obtained for water by Maneely (1962) and Neusen (1962) for various reservoir pressures and volume fractions. Note that the measured choked mass flow rates are bracketed by the homogeneous frozen and equilibrium curves and that the appropriately chosen partially frozen analysis is in close agreement with the experiments, despite the neglect (in the present model) of possible slip between the phases. The critical pressure ratio data is also in good agreement with the partially frozen analysis except for some discrepancy at the higher reservoir volume fractions.

It should be noted that the analytical approach described above is much simpler to implement than the numerical solution of the basic equations suggested by Henry and Fauske (1971). The latter does, however, have the advantage that slip between the phases was incorporated into the model.

Finally, information on the pressure, volume fraction, and velocity elsewhere in the duct (p/p_*, u/u_*, and α/α_*) as a function of the area ratio A/A_* follows from a procedure similar to that used for the noncondensable case in section (Nlf). Typical results for water with a reservoir pressure, p_o, of 500 *psia* and using the partially frozen analysis with $\epsilon_V = \alpha_o/2$ and $\epsilon_L = (1 - \alpha_o)/2$ are presented in figures 3 and 4. In comparing these results with those for the two-component mixture (figures 3 and 4, section (Nlf)) we observe that the pressure ratios are substantially smaller and do not vary monotonically with α_o. The volume fraction changes are smaller, while the velocity gradients are larger.