

Sonic Speed with Change of Phase

Turning now to the behavior of a two-phase rather than two-component mixture, it is necessary not only to consider the additional thermodynamic constraint required to establish the mass exchange, δm , but also to reconsider the two thermodynamic constraints, QA and QB , that were implicit in the two-component analysis of sections (Nlb), (Nlc), in the choice of the polytropic index, k , for the gas and the choice of the sonic speed, c_L , for the liquid. Note that a nonisentropic choice for k (for example, $k = 1$) implies that heat is exchanged between the components, and yet this heat transfer process was not explicitly considered, nor was an overall thermodynamic constraint such as might be placed on the global change in entropy.

We shall see that the two-phase case requires more intimate knowledge of these factors because the results are more sensitive to the thermodynamic constraints. In an ideal, infinitely homogenized mixture of vapor and liquid the phases would everywhere be in such close proximity to each other that heat transfer between the phases would occur instantaneously. The entire mixture of vapor and liquid would then always be in thermodynamic equilibrium. Indeed, one model of the response of the mixture, called the *homogeneous equilibrium model*, assumes this to be the case. In practice, however, there is a need for results for bubbly flows and mist flows in which heat transfer between the phases does not occur so readily. A second common model assumes zero heat transfer between the phases and is known as the *homogeneous frozen model*. In many circumstances the actual response lies somewhere between these extremes. A limited amount of heat transfer occurs between those portions of each phase that are close to the interface. In order to incorporate this in the analysis, we adopt an approach that includes the homogeneous equilibrium and homogeneous frozen responses as special cases but that requires a minor adjustment to the analysis of section (Nlb) in order to reflect the degree of thermal exchange between the phases. As in section (Nlb) ?? the total mass of the phases A and B after application of the incremental pressure, δp , are $\rho_A \alpha_A + \delta m$ and $\rho_B \alpha_B - \delta m$, respectively. We now define the fractions of each phase, ϵ_A and ϵ_B that, because of their proximity to the interface, exchange heat and therefore approach thermodynamic equilibrium with each other. The other fractions $(1 - \epsilon_A)$ and $(1 - \epsilon_B)$ are assumed to be effectively insulated so that they behave isentropically. This is, of course, a crude simplification of the actual circumstances, but it permits qualitative assessment of practical flows.

It follows that the volumes of the four fractions following the incremental change in pressure, δp , are

$$\begin{aligned} \frac{(1 - \epsilon_A)(\rho_A \alpha_A + \delta m)}{[\rho_A + \delta p(\partial \rho_A / \partial p)_s]}; & \quad \frac{\epsilon_A(\rho_A \alpha_A + \delta m)}{[\rho_A + \delta p(\partial \rho_A / \partial p)_e]} \\ \frac{(1 - \epsilon_B)(\rho_B \alpha_B - \delta m)}{[\rho_B + \delta p(\partial \rho_B / \partial p)_s]}; & \quad \frac{\epsilon_B(\rho_B \alpha_B - \delta m)}{[\rho_B + \delta p(\partial \rho_B / \partial p)_e]} \end{aligned} \quad (\text{Nld1})$$

where the subscripts s and e refer to isentropic and phase equilibrium derivatives, respectively. Then the change in total volume leads to the following modified form for equation (Nlb7) in the absence of surface tension:

$$\begin{aligned} \frac{1}{\rho c^2} = (1 - \epsilon_A) \frac{\alpha_A}{\rho_A} \left(\frac{\partial \rho_A}{\partial p} \right)_s + \epsilon_A \frac{\alpha_A}{\rho_A} \left(\frac{\partial \rho_A}{\partial p} \right)_e + (1 - \epsilon_B) \frac{\alpha_B}{\rho_B} \left(\frac{\partial \rho_B}{\partial p} \right)_s \\ + \epsilon_B \frac{\alpha_B}{\rho_B} \left(\frac{\partial \rho_B}{\partial p} \right)_e - \frac{\delta m}{\delta p} \left(\frac{1}{\rho_A} - \frac{1}{\rho_B} \right) \end{aligned} \quad (\text{Nld2})$$

The exchange of mass, δm , is now determined by imposing the constraint that the entropy of the whole be unchanged by the perturbation. The entropy prior to δp is

$$\rho_A \alpha_A s_A + \rho_B \alpha_B s_B \quad (\text{Nld3})$$

where s_A and s_B are the specific entropies of the two phases. Following the application of δp , the entropy is

$$(1 - \epsilon_A) \{ \rho_A \alpha_A + \delta m \} s_A + \epsilon_A \{ \rho_A \alpha_A + \delta m \} \{ s_A + \delta p (\partial s_A / \partial p)_e \} \\ + (1 - \epsilon_B) \{ \rho_B \alpha_B - \delta m \} s_B + \epsilon_B \{ \rho_B \alpha_B - \delta m \} \{ s_B + \delta p (\partial s_B / \partial p)_e \} \quad (\text{Nld4})$$

Equating (Nld3) and (Nld4) and writing the result in terms of the specific enthalpies h_A and h_B rather than s_A and s_B , one obtains

$$\frac{\delta m}{\delta p} = \frac{1}{(h_A - h_B)} \left[\epsilon_A \alpha_A \left\{ 1 - \rho_A \left(\frac{\partial h_A}{\partial p} \right)_e \right\} + \epsilon_B \alpha_B \left\{ 1 - \rho_B \left(\frac{\partial h_B}{\partial p} \right)_e \right\} \right] \quad (\text{Nld5})$$

Note that if the communicating fractions ϵ_A and ϵ_B were both zero, this would imply no exchange of mass. Thus $\epsilon_A = \epsilon_B = 0$ corresponds to the homogeneous frozen model (in which $\delta m = 0$) whereas $\epsilon_A = \epsilon_B = 1$ clearly yields the homogeneous equilibrium model.

Substituting equation (Nld5) into equation (Nld2) and rearranging the result, one can write

$$\frac{1}{\rho c^2} = \frac{\alpha_A}{p} [(1 - \epsilon_A) f_A + \epsilon_A g_A] + \frac{\alpha_B}{p} [(1 - \epsilon_B) f_B + \epsilon_B g_B] \quad (\text{Nld6})$$

where the quantities f_A , f_B , g_A , and g_B are purely thermodynamic properties of the two phases defined by

$$f_A = \left(\frac{\partial \ln \rho_A}{\partial \ln p} \right)_s \quad ; \quad f_B = \left(\frac{\partial \ln \rho_B}{\partial \ln p} \right)_s \quad (\text{Nld7}) \\ g_A = \left(\frac{\partial \ln \rho_A}{\partial \ln p} \right)_e + \left(\frac{1}{\rho_A} - \frac{1}{\rho_B} \right) \left(\rho_A h_A \frac{\partial \ln h_A}{\partial \ln p} - p \right)_e / (h_A - h_B) \\ g_B = \left(\frac{\partial \ln \rho_B}{\partial \ln p} \right)_e + \left(\frac{1}{\rho_A} - \frac{1}{\rho_B} \right) \left(\rho_B h_B \frac{\partial \ln h_B}{\partial \ln p} - p \right)_e / (h_A - h_B)$$

The sensitivity of the results to the, as yet, unspecified quantities ϵ_A and ϵ_B does not emerge until one substitutes vapor and liquid for the phases A and B ($A = V$, $B = L$, and $\alpha_A = \alpha$, $\alpha_B = 1 - \alpha$ for simplicity). The functions f_L , f_V , g_L , and g_V then become

$$f_V = \left(\frac{\partial \ln \rho_V}{\partial \ln p} \right)_s \quad ; \quad f_L = \left(\frac{\partial \ln \rho_L}{\partial \ln p} \right)_s \quad (\text{Nld8}) \\ g_V = \left(\frac{\partial \ln \rho_V}{\partial \ln p} \right)_e + \left(1 - \frac{\rho_V}{\rho_L} \right) \left(\frac{h_L}{\mathcal{L}} \frac{\partial \ln h_L}{\partial \ln p} + \frac{\partial \ln \mathcal{L}}{\partial \ln p} - \frac{p}{\mathcal{L} \rho_V} \right)_e \\ g_L = \left(\frac{\partial \ln \rho_L}{\partial \ln p} \right)_e + \left(\frac{\rho_L}{\rho_V} - 1 \right) \left(\frac{h_L}{\mathcal{L}} \frac{\partial \ln h_L}{\partial \ln p} - \frac{p}{\mathcal{L} \rho_L} \right)_e$$

where $\mathcal{L} = h_V - h_L$ is the latent heat. It is normally adequate to approximate f_V and f_L by the reciprocal of the ratio of specific heats for the gas and zero respectively. Thus f_V is of order unity and f_L is very small. Furthermore g_L and g_V can readily be calculated for any fluid as functions of pressure or temperature. Some particular values are shown in figure 1. Note that g_V is close to unity for most fluids except in the neighborhood of the critical point. On the other hand, g_L can be a large number that varies considerably with pressure. To a first approximation, g_L is given by $g^*(p_C/p)^\eta$ where p_C is the critical pressure and, as indicated in figure 1, g^* and η are respectively 1.67 and 0.73 for water. Thus, in summary, $f_L \approx 0$, f_V and g_V are of order unity, and g_L varies significantly with pressure and may be large.

With these magnitudes in mind, we now examine the sensitivity of $1/\rho c^2$ to the interacting fluid fractions ϵ_L and ϵ_V :

$$\frac{1}{\rho c^2} = \frac{\alpha}{p} [(1 - \epsilon_V) f_V + \epsilon_V g_V] + \frac{(1 - \alpha)}{p} \epsilon_L g_L \quad (\text{N1d9})$$

Using $g_L = g^*(p_c/p)^\eta$ this is written for future convenience in the form:

$$\frac{1}{\rho c^2} = \frac{\alpha k_V}{p} + \frac{(1 - \alpha) k_L}{p^{1+\eta}} \quad (\text{N1d10})$$

where $k_V = (1 - \epsilon_V) f_V + \epsilon_V g_V$ and $k_L = \epsilon_L g^*(p_c)^\eta$. Note first that the result is rather insensitive to ϵ_V since f_V and g_V are both of order unity. On the other hand $1/\rho c^2$ is sensitive to the interacting liquid fraction ϵ_L though this sensitivity disappears as α approaches 1, in other words for mist flow. Thus the choice of ϵ_L is most important at low vapor volume fractions (for bubbly flows). In such cases, one possible qualitative estimate is that the interacting liquid fraction, ϵ_L , should be of the same order as the gas volume fraction, α . In section (N1g) we will examine the effect of the choice of ϵ_L and ϵ_V on a typical vapor/liquid flow and compare the model with experimental measurements.

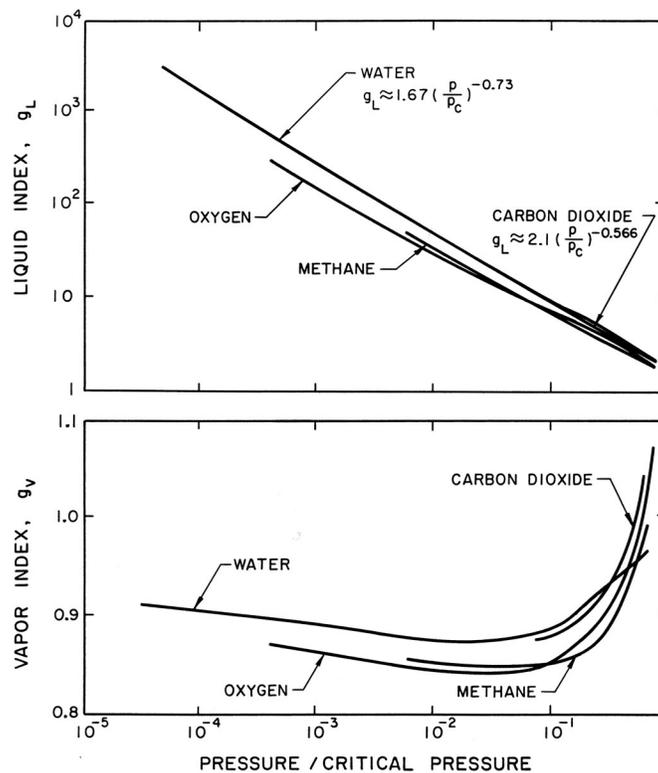


Figure 1: Typical values of the liquid index, g_L , and the vapor index, g_V , for various fluids.