Sonic Speed

Consider an infinitesimal volume of a mixture consisting of a disperse phase denoted by the subscript A and a continuous phase denoted by the subscript B. For convenience assume the initial volume to be unity. Denote the initial densities by ρ_A and ρ_B and the initial pressure in the *continuous* phase by p_B. Surface tension, S, can be included by denoting the radius of the disperse phase particles by R. Then the initial pressure in the disperse phase is $p_A = p_B + 2S/R$.

Now consider that the pressure, p_A, is changed to $p_A + \delta p_A$ where the difference δp_A is infinitesimal. Any dynamics associated with the resulting fluid motions will be ignored for the moment. It is assumed that a new equilibrium state is achieved and that, in the process, a mass, δm, is transferred from the continuous to the disperse phase. It follows that the new disperse and continuous phase masses are $\rho_A\alpha_A + \delta m$ and $\rho_B\alpha_B - \delta m$ respectively where, of course, $\alpha_B = 1 - \alpha_A$. Hence the new disperse and continuous phase volumes are respectively

$$(\rho_A\alpha_A + \delta m) \Big/ \left[\rho_A + \frac{\partial \rho_A}{\partial p_A}\Big|_{QA}\delta p_A\right] \tag{Nlb1}$$

and

$$(\rho_B\alpha_B - \delta m) \Big/ \left[\rho_B + \frac{\partial \rho_B}{\partial p_B}\Big|_{QB}\delta p_B\right] \tag{Nlb2}$$

where the thermodynamic constraints QA and QB are, as yet, unspecified. Adding these together and subtracting unity, one obtains the change in total volume, δV, and hence the sonic velocity, c, as

$$c^{-2} = -\rho \frac{\delta V}{\delta p_B}\Big|_{\delta p_B \to 0} \tag{Nlb3}$$

$$c^{-2} = \rho\left[\frac{\alpha_A}{\rho_A}\;\frac{\partial \rho_A}{\partial p_A}\Big|_{QA}\frac{\delta p_A}{\delta p_B} + \frac{\alpha_B}{\rho_B}\;\frac{\partial \rho_B}{\partial p_B}\Big|_{QB} - \frac{(\rho_B - \rho_A)}{\rho_A\rho_B}\;\frac{\delta m}{\delta p_B}\right] \tag{Nlb4}$$

If it is assumed that no disperse particles are created or destroyed, then the ratio $\delta p_A/\delta p_B$ may be determined by evaluating the new disperse particle size $R + \delta R$ commensurate with the new disperse phase volume and using the relation $\delta p_A = \delta p_B - \frac{2S}{R^2}\delta R$:

$$\frac{\delta p_A}{\delta p_B} = \left[1 - \frac{2S}{3\alpha_A\rho_A R}\;\frac{\delta m}{\delta p_B}\right] \Big/ \left[1 - \frac{2S}{3\rho_A R}\;\frac{\partial \rho_A}{\partial p_A}\Big|_{QA}\right] \tag{Nlb5}$$

Substituting this into equation (Nlb4) and using, for convenience, the notation

$$\frac{1}{c_A^2} = \frac{\partial \rho_A}{\partial p_A}\Big|_{QA} \quad ; \quad \frac{1}{c_B^2} = \frac{\partial \rho_B}{\partial p_B}\Big|_{QB} \tag{Nlb6}$$

the result can be written as

$$\frac{1}{\rho c^2} = \frac{\alpha_B}{\rho_B c_B^2} + \frac{\left[\frac{\alpha_A}{\rho_A c_A^2} - \frac{\delta m}{\delta p_B}\left\{\frac{1}{\rho_A} - \frac{1}{\rho_B} + \frac{2S}{3\rho_A\rho_B c_A^2 R}\right\}\right]}{\left[1 - \frac{2S}{3\rho_A c_A^2 R}\right]} \tag{Nlb7}$$

This expression for the sonic speed, c, is incomplete in several respects. First, appropriate thermodynamic constraints QA and QB must be identified. Second, some additional constraint is necessary to establish the

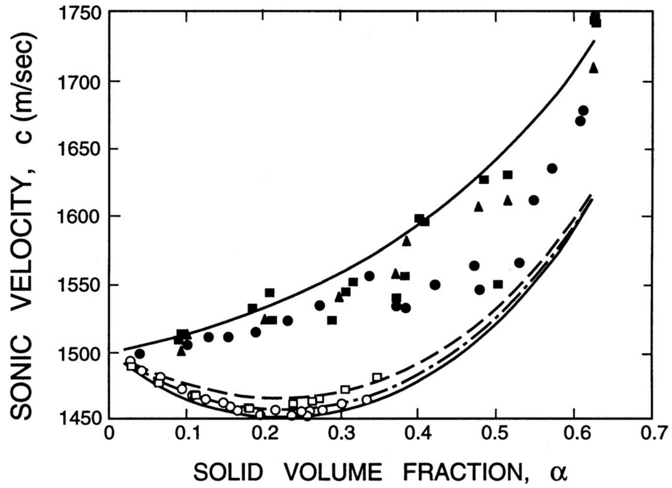

Figure 1: The sonic velocities for various suspensions of particles in water: ⊙, frequency of $100kHz$ in a suspension of $1\mu m$ Kaolin particles (Hampton 1967) ($2\kappa R = 6.6 \times 10^{-5}$); ⊡, frequency of $1MHz$ in a suspension of $0.5\mu m$ Kaolin particles (Urick 1948) ($2\kappa R = 3.4 \times 10^{-4}$); solid symbols, frequencies of $100kHz - 1MHz$ in a suspension of $0.5mm$ silica particles (Atkinson and Kytömaa 1992) ($2\kappa R = 0.2 - 0.6$). Lines are theoretical predictions for $2\kappa R = 0$, 6.6×10^{-5}, 3.4×10^{-4}, and $2\kappa R = 0.2 - 0.6$ in ascending order (from Atkinson and Kytömaa 1992).

relation $\delta m/\delta p_B$. But before entering into a discussion of appropriate practical choices for these constraints (see section (Nld)) several simpler versions of equation (Nlb7) should be identified.

First, in the absence of any exchange of mass between the components the result (Nlb7) reduces to

$$\frac{1}{\rho c^2} = \frac{\alpha_B}{\rho_B c_B^2} + \frac{\frac{\alpha_A}{\rho_A c_A^2}}{\left\{1 - \frac{2S}{3\rho_A c_A^2 R}\right\}} \tag{Nlb8}$$

In most practical circumstances the surface tension effect can be neglected since $S \ll \rho_A c_A^2 R$; then equation (Nlb8) becomes

$$\frac{1}{c^2} = \{\rho_A \alpha_A + \rho_B \alpha_B\} \left[\frac{\alpha_B}{\rho_B c_B^2} + \frac{\alpha_A}{\rho_A c_A^2}\right] \tag{Nlb9}$$

In other words, the acoustic impedance for the mixture, namely $1/\rho c^2$, is simply given by the average of the acoustic impedance of the components weighted according to their volume fractions. Another popular way of expressing equation (Nlb9) is to recognize that ρc^2 is the effective bulk modulus of the mixture and that the inverse of this effective bulk modulus is equal to an average of the inverse bulk moduli of the components ($1/\rho_A c_A^2$ and $1/\rho_B c_B^2$) weighted according to their volume fractions.

Some typical experimental and theoretical data obtained by Hampton (1967), Urick (1948) and Atkinson and Kytömaa (1992) is presented in figure 1. Each set is for a different ratio of the particle size (radius, R) to the wavelength of the sound (given by the inverse of the wavenumber, κ). Clearly the theory described above assumes a continuum and is therefore relevant to the limit $\kappa R \rightarrow 0$. The data in the figure shows good agreement with the theory in this low frequency limit. The changes that occur at higher frequency (larger κR) will be discussed in the next section.

Perhaps the most dramatic effects occur when one of the components is a gas (subscript G), that is much more compressible than the other component (a liquid or solid, subscript L). In the absence of surface tension ($p = p_G = p_L$), according to equation (Nlb9), it matters not whether the gas is the continuous

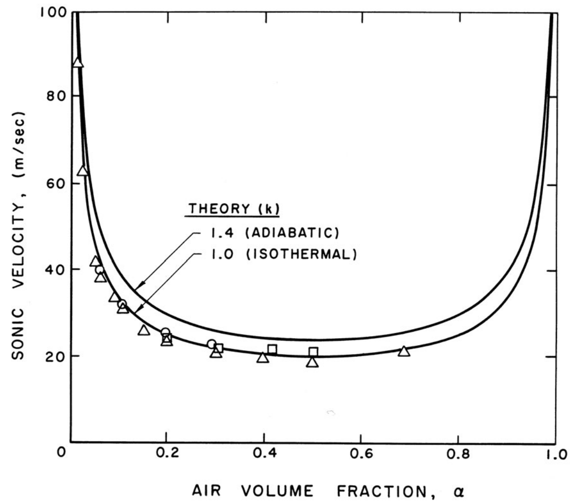

Figure 2: The sonic velocity in a bubbly air/water mixture at atmospheric pressure for $k = 1.0$ and 1.4. Experimental data presented is from Karplus (1958) and Gouse and Brown (1964) for frequencies of 1 kHz ($\odot$), 0.5 kHz ($\boxdot$), and extrapolated to zero frequency($\triangle$).

or the disperse phase. Denoting α_G by α for convenience and assuming the gas is perfect and behaves polytropically according to $\rho_G^k \propto p$, equation (Nlb9) may be written as

$$\frac{1}{c^2} = [\rho_L(1-\alpha) + \rho_G\alpha]\left[\frac{\alpha}{kp} + \frac{(1-\alpha)}{\rho_L c_L^2}\right] \qquad \text{(Nlb10)}$$

This is the familiar form for the sonic speed in a two-component gas/liquid or gas/solid flow. In many applications $p/\rho_L c_L^2 \ll 1$ and hence this expression may be further simplified to

$$\frac{1}{c^2} = \frac{\alpha}{kp}[\rho_L(1-\alpha) + \rho_G\alpha] \qquad \text{(Nlb11)}$$

Note however, that this approximation will not hold for small values of the gas volume fraction α.

Equation (Nlb10) and its special properties were first identified by Minnaert (1933). It clearly exhibits one of the most remarkable features of the sonic velocity of gas/liquid or gas/solid mixtures. The sonic velocity of the mixture can be very much smaller than that of either of its constituents. This is illustrated in figure 2 where the speed of sound, c, in an air/water bubbly mixture is plotted against the air volume fraction, α. Results are shown for both isothermal ($k = 1$) and adiabatic ($k = 1.4$) bubble behavior using equation (Nlb10) or (Nlb11), the curves for these two equations being indistinguishable on the scale of the figure. Note that sonic velocities as low as 20 m/s occur.

Also shown in figure 2 is experimental data of Karplus (1958) and Gouse and Brown (1964). Data for frequencies of 1.0 kHz and 0.5 kHz are shown in figure 2, as well as data extrapolated to zero frequency. The last should be compared with the low frequency analytical results presented here. Note that the data corresponds to the isothermal theory, indicating that the heat transfer between the bubbles and the liquid is sufficient to maintain the air in the bubbles at roughly constant temperature.

Further discussion of the acoustic characteristics of dusty gases is presented later in section (Nnf) where the effects of relative motion between the particles and the gas are included. Also, the acoustic characteristics of dilute bubbly mixtures are further discussed in section (Nmc) where the dynamic response of the bubbles are included in the analysis.