Wave Propagation

The wave propagation characteristics of conventional forms of matter are well understood and well documented. Waves in granular materials are more complex due to the mechanics of the force chains that transmit the stress and therefore the waves. These nonlinear chains are prone to continual rearrangement even by the slightest of forces.

Wave Propagation in Static Granular Beds:

Static granular materials exhibit unusual wave propagation characteristics because of the nonlinear nature of the contacts between the particles that form the force chains and therefore propagate the waves. Simulations (Haff 1987) and experiments (Liu 1994) have both shown the susceptibility of force chains to rearrangement. In the work of Liu and Nagel (1993), disturbances as weak as the thermal expansion of a particle far from the measurement point or the input of the pressure wave itself was found to change the wave propagation characteristics of the granular bed. Consequently there is great scatter in the measurements of wave speeds in static granular beds. On the other hand, in a flowing granular material the chains reorganize constantly. (Note: Another type of wave is possible in flowing granular systems, namely waves in the particle concentration, known as kinematic or continuity waves, as discussed in Section (Ns)). Past experiments on wave propagation in granular materials have explored numerous experimental conditions and have used a variety of techniques to determine the wave speed. Not surprisingly, there is a wide range of reported wave speeds. One of the more important variables in the experiments is the pressure at which the wave speed measurement is made. Consequently the reported wave speeds vary over more than an order of magnitude from 50 to $1350 m/s$.

Duffy and Mindlin (1957) used the excitation of a resonant column of granular material (ball bearings packed into a face-centered cubic lattice) to measure the wave speed of that granular material. The packing was enclosed in a rubber bag and the bag was evacuated to varying degrees, leading to range of isostatic confining pressures. They found that the wave speed, c, scaled with pressure, p, like $c \propto p^{1/6}$ (as their theory predicted) for pressures above $69 kPa$. Their theory assumed particles of identical diameter and they explained a departure in the scaling of the experimental measurements at lower pressures on a reduced number of particle contacts (the coordination number) as due to variations in the particle diameters. Hardin and Richart (1963) applied the same resonance method to a random packing of Ottawa sand. In addition to the confining pressure, they examined the effect on the wave speed of void fraction and water saturation. They found the wave speed scaled like $c \propto p^{1/4}$ for both wet and dry sand.

Static granular materials also manifest interesting behavior due to the nonlinear nature of the contacts between the particles that form th force chains. In his analysis of the low pressure behaviors, Goddard (1990) found that for Hertzian contacts, the scaling of the wave speed with pressure should be $c \propto p^{1/6}$. He suggested two mechanisms for the quarter-power scaling observed by Duffy and Mindlin (1957) at relatively low pressure. One explanation was based on a non-Hertzian contact at low pressures where, Goddard argued, conical asperities dominate the interaction between particles. Aa a second explanation, Goddard suggested that the coordination number will increase as the pressure increases. Both mechanisms were shown to display a 1/4 power scaling at lower pressure and a 1/6 power scaling at higher pressure. Granular dynamics simulations by Makse *et al.* (1999) supported the coordination number argument for the pressure scaling transition. Their simulation results for the bulk modulus, and thus the compressional wave speed, agreed well with analytic calculations when the increase in the coordination number with increasing pressure is taken into account. Velicky and Caroli (2002) propose yet another explanation for the pressure scaling difference; they attribute the quarter-power scaling to disorder-induced stress fluctuations in the granular bed.

The unusual characteristics of wave propagation in macroscopic granular materials were also explored by Liu and Nagel (1993). The dependence of the propagation on a limited number of wave paths (chains) was explored by using $5mm$ glass spheres. Measurements were made with an accelerometer of comparable size to an individual grain at distances as close as $4cm$ from the wave source. Such an arrangement means that as few as 8 particles separate the source and detector. The imposed pressure at the point of measurement was minimal as it was solely due to the weight of the overlaying particles of depth $6cm$. The authors used two techniques to calculate the wave propagation in the granular material and found two very different speeds. The time-of-flight measurement used the time required for a pulse introduced at the source to be detected at the buried accelerometer. In contrast, the group velocity measurement used a sinusoidal input at the wave source. The phase shift between the source and detected signals was found for a range of input frequencies. The group velocity was calculated from the slope of the resulting linear trend. The speeds from the two methods differed by about a factor of five though, for both techniques, there was a wide scatter in the measurements.

Jia *et al.* (1999) attempted to address the discrepancy in the wave speeds found by Liu and Nagel (1993). They used ultrasonic wave pulses to explore wave propagation in a frequency range passing through the effective wavelength that is comparable to the grain size. By doing this, they identified two distinct waves, which they term E and S waves. The E wave is a coherent, self-averaging wave with a narrow band of relatively low frequencies. Its effective wavelength is much larger than the particle size. They found that the speed of this wave scaled with pressure like $c \propto p^{1/4}$ at lower pressures and $c \propto p^{1/6}$ at higher pressures. In contrast, the S wave had an irregular structure with an irregular, high frequency spectrum. This higher frequency signal had an effective wavelength comparable to the system grain size so it was argued that the irregular character of this wave was due to multiple scattering and interference in the granular material.

One application of granular materials is their use to mitigate shocks resulting from explosions. To this end, Ben-Dor *et al.* (1997) studied the interaction of a shock wave with a granular bed. They created a granular bed at the end of a long shock tube. The impact of the shock led to a series of waves in the bed. They identified each wave (transmitted, compaction, and weak rarefaction wave) from pressure traces.

Hostler (2004) and Hostler and Brennen (2005 and 2005a) also conducted experiments and simulations on wave propagation using transducer measurements at two points in a granular bed to measure the transmission on waves generated by a piston at one end of the bed (Figure 1). The experiments revealed unique

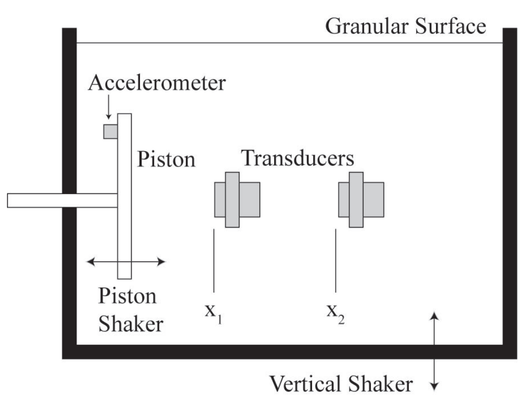

Figure 1: Schematic of wave propagation experiments. From Hostler and Brennen (2005).

features of waves in granular systems that result from the nonlinearity of the bed and the heterogeneity and ephemerality of the force chains. Sinusoidal wave excitation demonstrated the nondispersive nature of the waves and showed the transient effects of force chain rearrangement. Single-pulse waves displayed a semi-permanent shape qualitatively similar to predictions from the nonlinear theory of one-dimensional chains (see below and Nesterenko 1984, 2001).

Two different forms of excitation were used to produce waves. The first, a sinusoidal piston motion, allowed for the examination of the dispersion characteristics of the granular bed and the variability of measurements of the phase speed and attenuation with time. The second was a pulsed movement of the piston that created a wave of finite duration that provides information about the nonlinear nature of wave propagation in the granular bed. Experiments and complimentary simulations were performed for both forms of wave input.

The sinusoidal input studies were divided into two sets of experiments. The first set fixed the acceleration amplitude of the piston and varied the frequency. These experiments showed that the wave propagation was nondispersive as was previously observed by Liu and Nagel (1994). Since the propagation was nondispersive, the group velocity (which is a constant for nondispersive waves) was used to compare different experimental conditions. The greatest influence on the group velocity was found to be the material composition of the particles as characterized by the sound speed ($(E/\rho)^(1/2)$) in the bulk material. The group velocity was measured to be $170m/s$in glass beads and $70m/s$ in PVC cylinders; these velocities were consistent with Hardin and Richart ($100-200m/s$) though larger than Liu and Nagel ($50-90m/s$). The effect of particle size was minimal. Consolidation and an increased overburden of particles increased the group velocity for the glass particles between $10-20\%$. Measurements of the attenuation showed an increasing trend with increasing frequency in an irregular fashion. The attenuation changed sharply with small changes to the frequency. Again, particle composition was found to have the largest effect.

Simulations with a sinusoidal input agreed well with the experiments provided the particle/particle interaction model was realistic. The simulated pressure signals were qualitatively similar to the experimental traces and the simulations also showed that the wave propagation was nondispersive. Group velocity measurements in the simulations resulted in higher values ($252-277m/s$ for glass particles). This mismatch with experiments indicates the need for some adjustment in the particle contact model. The simulations also displayed system resonances that were seen in the experiments.

The second set of sinusoidal input experiments involved fixing the piston frequency and varying the acceleration amplitude. These constant frequency experiments demonstrated the tenuous nature of the particle contact network. Measurements of the phase speed and attenuation in an unconsolidated bed showed a high degree of hysteresis while increasing and then decreasing the acceleration amplitude of the piston (see Figures 2 and 3). Scatter was also high in these measurements as both the phase speed and attenuation varied with time at large input amplitudes. Both the scatter and the hysteresis are the result of particle and force chain rearrangement from the action of the input waves alone. Performing the same experiments in a consolidated bed led to more repeatable data and reduced scatter.

The second form of input into the granular bed was a pulsed wave. Such finite duration waves are particularly useful for probing the attenuation characteristics of the bed since the energy input is finite. The amplitude of these waves was seen to decrease exponentially with distance and the rate of decay was 2.5 times greater in the PVC particles than in the glass particles.

Two sets of experiments were performed to elucidate the relationship between the input pulse shape and the characteristics of the measured wave. In the first set, the amplitude of the input pulse amplitude was fixed and the pulse width was varied. A semi-permanent wave regime was observed for the smallest pulse widths. In this range, the detected width was $10ms$ regardless of changes to the input width (see

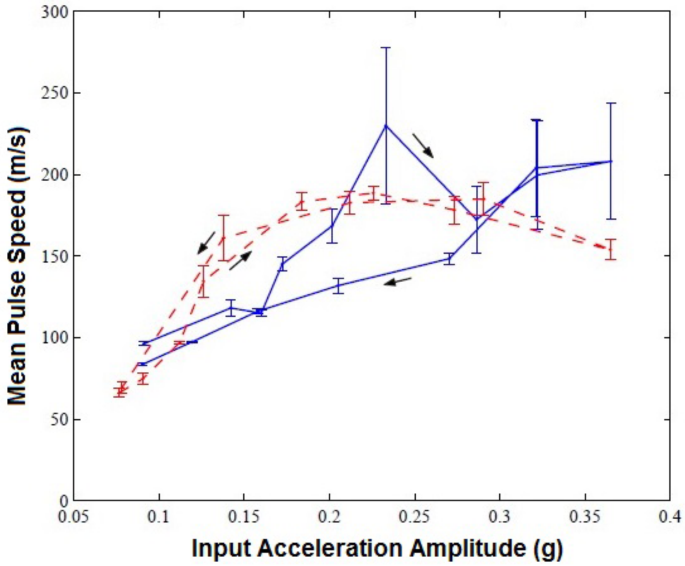

Figure 2: Mean phase speed for an unconsolidated bed (solid line) and a consolidated bed (dashed line) of $4mm$ glass beads as the amplitude of the input wave is increased and then decreased. From Hostler (2004) and Hostler and Brennen (2005).

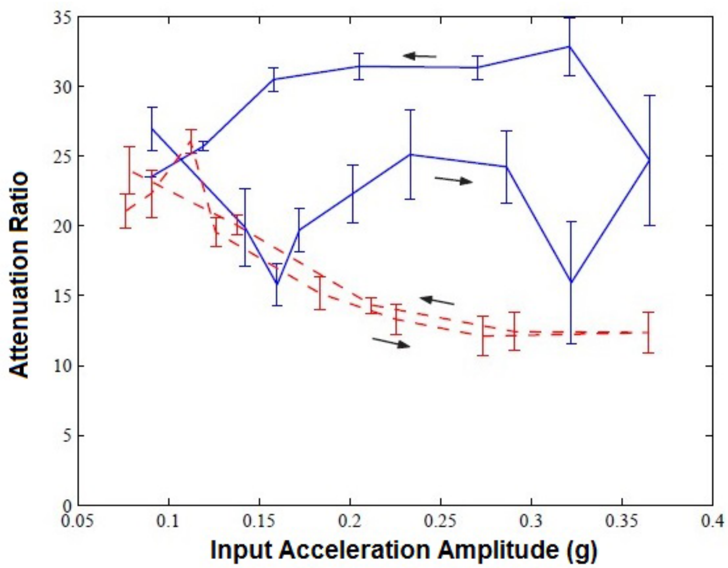

Figure 3: Attenuation ratio for an unconsolidated bed (solid line) and a consolidated bed (dashed line) of $4mm$ glass beads as the amplitude of the input wave is increased and then decreased. From Hostler (2004) and Hostler and Brennen (2005).

for example Figure 6). These *solitary waves* were an unexpected finding. At the other extreme, both the amplitude of the waves and the attenuation were constant at sufficiently large input widths. Typical data are shown in Figures 4 and 5).

Constant pulse width experiments confirmed previous observations on the effect of increasing wave amplitude. Particle rearrangement led to increased scatter and irregular changes in wave amplitude and width measurements for the largest input amplitudes. Within the data scatter, there was no variation of the wave speed with wave amplitude. The wave speeds measured in both the glass ($120m/s$) and PVC ($60m/s$) particles were comparable to, but slightly smaller than, the group velocities measured in the continuous input experiments. Consolidation increased the wave speed from 140 to $175m/s$ in the glass particles and from 60 to $80m/s$ in the PVC particles (see Figure 7).

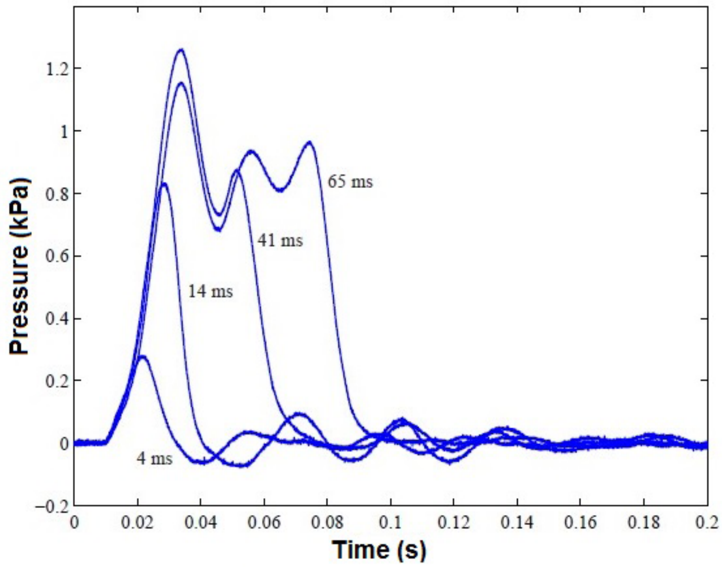

Figure 4: Wave shape in a bed of PVC particles for four different input pulse widths (4, 14, 41, 65ms) for a fixed input pulse amplitude and fixed distance from the pulse generator. From Hostler (2004) and Hostler and Brennen (2005).

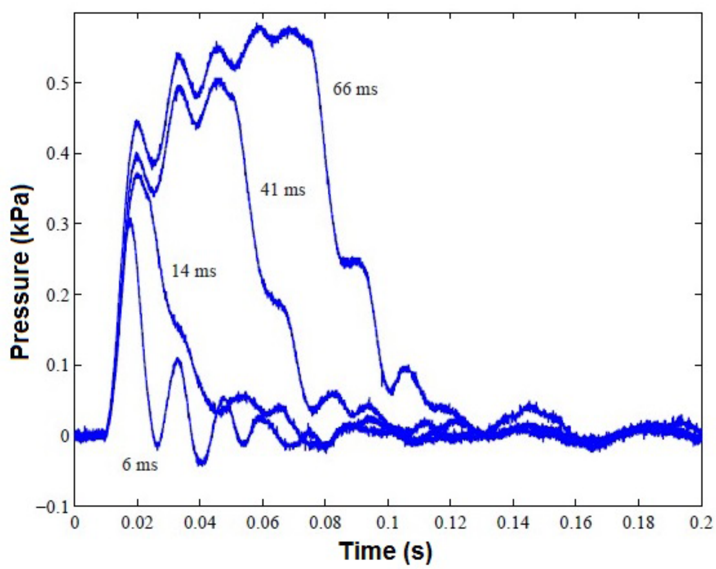

Figure 5: Wave shape in a bed of 3mm glass beads for four different input pulse widths (6, 14, 41, 66ms) for a fixed input pulse amplitude and fixed distance from the pulse generator. From Hostler (2004) and Hostler and Brennen (2005).

Some interesting oscillations in the pressure signal occurred inside the primary pulsed waves. Due to the relatively low frequency ($40-80Hz$) of these oscillations, they must occur over some appreciable length and may be the result of a natural frequency of the force chains. To support this idea, the oscillation frequency was compared to the confining pressure. Consistent with what is expected for the natural frequency of a force chain, the oscillation frequency increased for larger confining pressure, increased with the stiffness of the particle material and decreased as the length of the chain increased. Analysis of a one-dimensional linear chain of particles showed that the natural frequency would be of the correct order for a chain of reasonable length (100 particles).

Computer Simulation of Wave Propagation in Static Granular Beds:

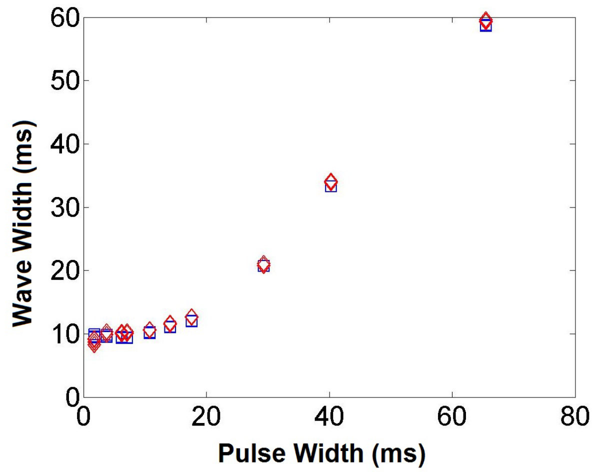

Figure 6: Shock width in PVC particles as a function of the input pulse width. From Hostler and Brennen (2005).

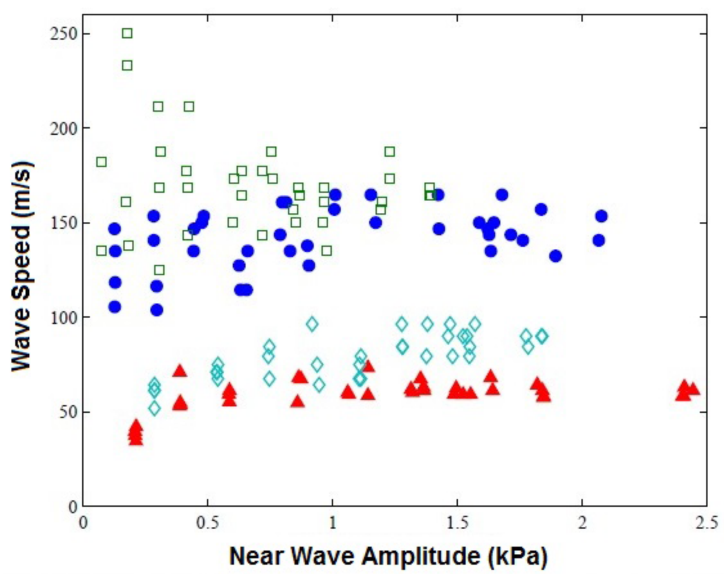

Figure 7: Wave speeds in beds of $2mm$ glass and PVC beads for both unconsolidated (PVC (△), glass (•)) and consolidated (PVC (◇), glass (□)) as a function of wave amplitude. From Hostler (2004).

Results from the simulations with a pulsed wave input showed good agreement with the experiments and allowed for even greater exploration of the characteristics of these waves. Over a range of parameter values, the pressure traces from the simulations matched the wave shapes seen in the experiments. The exceptions occurred at the extremes of the values describing the input pulse shape. For the largest input amplitudes in the simulations, shock-like steepening is observed. Nothing like this was seen in the experiments.

One advantage of the simulations is the ability to visualize the wave at any point in the granular bed. The experiments were limited to measurements of the pressure at a few points in the bed. In the simulations, the force condition at any point in the bed is known. Using this knowledge, the wave can be imaged as it is transmitted between particle contacts. Such visualizations show the structure of the wave and how it varies with time and depth in the bed. Diffusion of the wave energy can be seen as the wave traverses the bed. This diffusion coupled with nonconservative particle interactions leads to a net dissipation that is evident in the pressure traces at the far wall of the simulation cell.

Simulations also allowed for a wider variation of the system parameters, particularly the material properties of the particles. Sensitivity studies on the wave speed, revealed a strong and unexpected dependence on Poisson's ratio of the particles. The dissipation factor, the pulse amplitude, and the pulse width were also found to effect the wave speed, but to a lesser degree. The confining pressure (as dictated by the depth in the bed at which the measurement was made) was also found to strongly influence the wave speed (see also Goddard 1990).

Wave Propagation in Shaken Granular Beds:

Hostler (2004) (see also Hostler and Brennen (2005a)) found that measurements of the wave characteristics in a shaken granular bed were possible even for substantial amplitudes of shaking. The imposed shaking actively rearranges the force chains through which the waves are propagated. The prevailing confining pressure, which naturally changes throughout a shaking cycle, was determined to be the system parameter that correlates best with changes to the wave speed. Experiments with both a sinusoidal and a pulsed wave input showed that elastic waves could be detected and their properties determined. Although pressure traces included the effects of the shaking as well as the wave input, these contributions could be filtered out. In the continuous input cases, extraction of the wave input signal required a sufficient separation of the frequencies of the wave source and the agitation. Filtering was then used to remove the component due to the shaking. In the pulsed wave cases, the repeatability of the pressure due to the shaking allowed for a specially devised subtraction procedure. Simulations displayed the same superposition of the pressure components from the wave source and shaking and the signals were qualitatively similar to the experiments.

The spectra of the signals in the sinusoidal input experiments demonstrated the nonlinear nature of wave propagation in the shaken granular bed. In addition to peaks from the input wave and shaking frequencies, beat frequencies were observed at increments of the shaking frequency about the input wave frequency. The number of beat frequencies increased with the shaking amplitude. Such beating is indicative of a quadratic nonlinearity in the system. The corresponding simulations showed identical behavior in their spectra.

Phase speed measurements for the sinusoidal input experiments showed interesting behavior as the agitation amplitude was increased (see Figure 8). For low shaking amplitudes, the phase speed took different values after undergoing an increasing then decreasing shaking cycle. Differences in the measured speed could be as great as $50m/s$. At sufficiently high shaking amplitudes the phase speed became single-valued. In this range, it appeared that the phase speed was governed more by the granular state rather than the geometrical configuration of the bed. The transition to this single-valued regime occurred at a different shaking amplitude for the glass spheres than the PVC cylinders.

In a shaken bed, the parameter that most effects the wave speed is the local confining pressure. As observed by Potapov and Campbell (1996), the solids fraction is a weak indicator of changes in the contact condition between particles. A significant change to the bed microstructure is required to significantly influence the solids fraction. In contrast, imperceptible changes in the microstructure were seen to effect wave propagation (Haff 1987, Liu and Nagel 1993). The local confining pressure is a much better predictor of changes at the particle contacts.

Simulations of Wave Propagation in a Shaken Granular Bed:

Hostler (2004) (see also Hostler and Brennen (2005a)) also performed simulations that were a two-dimensional version of the experiments and used a discrete, soft-particle method to detect the wave at both the output of the simulated bed and at any point within it. In addition to examining the wave propagation in the granular bed at rest, simulations and experiments were also performed for a granular bed undergoing shaking perpendicular to the direction of the wave input. Imposed shaking increased the

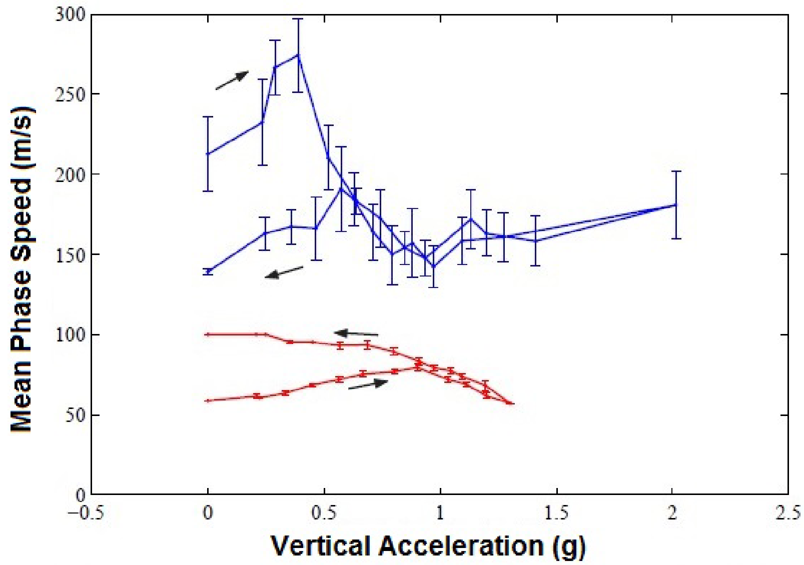

Figure 8: Phase speeds in a shaken bed of $4mm$ glass beads (blue) and PVC particles (red) as the shaking amplitude is first increased and then decreased. From Hostler (2004).

granular temperature of the bed and allowed for the exploration of the effect of granular state changes on the wave propagation characteristics.

Simulations with sinusoidal input did show the temporal variation of the granular bed state at various intervals of the shaking. The input wave could only be detected during portions of the agitation period. At times when the bed was locally expanded, zero pressure was detected. During high pressure events, such as the collision of the bed floor with the granular bed, the input source was lost in this noisy pressure spike. The input wave signal was only evident at times when the bed was sufficiently compressed.

Simulations with a pulsed input further examined the effect of the state of the granular bed on wave

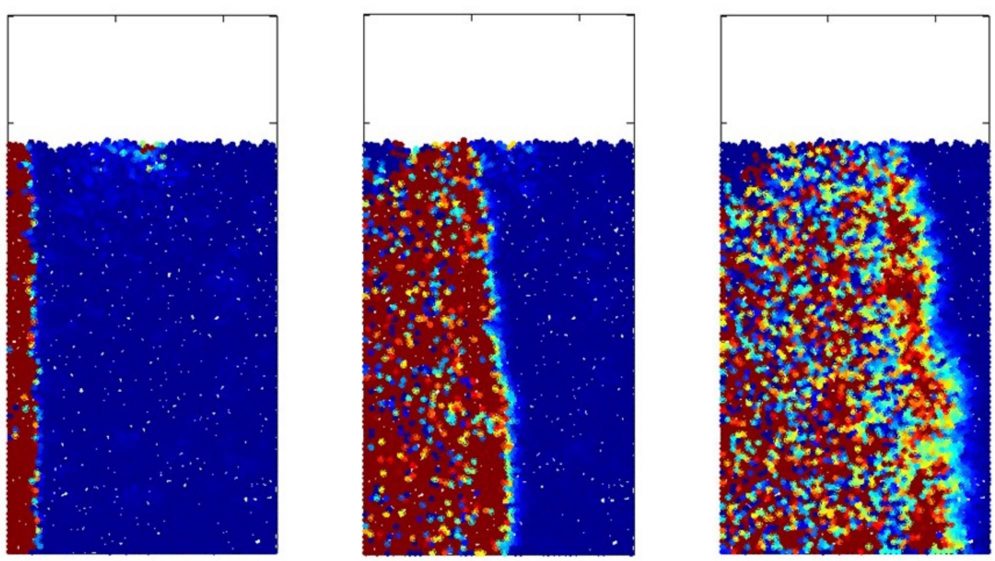

Figure 9: Transmission of a pressure wave in a static granular bed for three consecutive moments in time. From Hostler (2004).

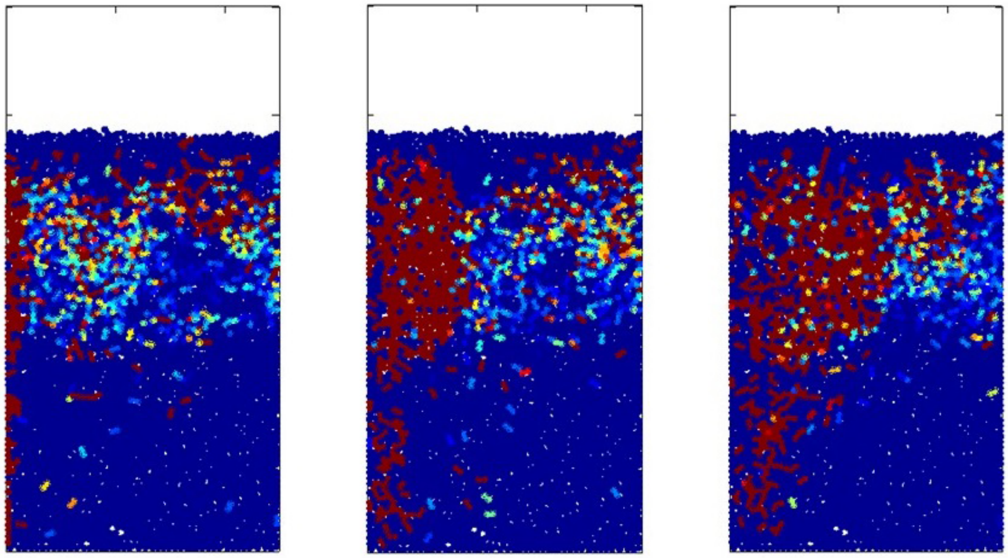

Figure 10: Transmission of a pressure wave in a shaken granular bed for three consecutive moments in time. The three images were captured at a moment in the shaking period when the upper particle layers are compressed while the lower particle layers are loose. From Hostler (2004).

propagation properties through it. Pressure traces and visualizations of forces within the bed showed that due to state changes induced by the shaking, wave propagation could be spatially localized. Propagation was possible in the upper layers of the bed where the bed was in a relatively compressed state. On the other hand, the expanded state of the lower portion of the granular bed prevented elastic wave propagation. In the region where propagation was possible, the wave speed was found to be lower than that measured in an non-shaken bed. Two sample simulations are depicted in Figures 9 and 10. The particle bed is first formed by sedimentation; the wave is then created by displacing the lefthand wall of the box. In the case of the shaken bed (Figure 10) the three images were captured at a moment in the shaking period when the upper particle layers are compressed while the lower particle layers are loose.

Dissipation of Waves in a Granular Bed:

The body of work on the dissipation of waves in granular materials is quite limited. There are a number of mechanisms through which wave energy is dissipated in a granular bed. Since particle contacts are inelastic and frictional, some energy is lost during particle interactions. Frictional effects convert some of the wave energy into heat. Plastic deformation of the particles may also absorb some of the wave energy. A second loss mechanism involves particle rearrangement. Moreover some energy may be converted to kinetic energy of a particle when it breaks loose from its neighbors. A third possible mechanism is the scattering of wave energy through the particle contact network. Wave energy can scatter away from the point at which measurements are made giving the appearance that the energy has been lost rather than been redirected.

Addendum: One Dimensional Chains:

Nesterenko (2001) showed that for a one-dimensional, linear chain of particles, nonlinear wave solutions exist in two limits. One limit is that in which particle displacements are very small compared to the initial overlap due to the confining force. Nesterenko termed this situation the strongly-compressed chain. The other limit, the weakly compressed chain, is the opposite case in which the particle displacement is large or comparable to the initial overlap. Nesterenko (1984) showed that non-linear, solitary wave solutions exist for both cases and Lazaridi and Nesterenko (1985) and Coste *et al.* (1997) observed solitary waves experimentally. Typically the width of the solitary waves in a one-dimensional chain is about five particle diameters, much narrower than the solitary waves in a two- or three-dimensional granular medium.