Slow Granular Flow

All of the early efforts to understand granular flow neglected the random kinetic energy of the particles, the granular temperature, and sought to construct equations for the motion as extrapolations of the theories of soil mechanics by including the mean or global inertial effects in the equations of motion. We now recognize that, if these constructs are viable, they apply to the elastic-quasistatic regime of slow granular motion. Notable among these theories were those who sought to construct effective continuum equations of motion for the granular material beginning with

$$\frac{D(\rho_S \alpha)}{Dt} + \rho_S \alpha \frac{\partial u_i}{\partial x_i} = 0 \tag{Npg1}$$

$$\rho_S \alpha \frac{D u_k}{Dt} = \rho_S \alpha g_k - \frac{\partial \sigma_{ki}}{\partial x_i} \tag{Npg2}$$

where equation (Npg1) is the continuity equation (Nbb6) and equation (Npg2) is the momentum equation (equation (Nbe9) for a single phase flow). It is then assumed that the stress tensor is quasistatic and determined by conventional soil mechanics constructs. A number of models have been suggested but here we will focus on the most commonly used approach, namely Mohr-Coulomb models for the stresses.