Basic Equations

First we review the fundamental equations governing the flow of the individual phases or components in a dusty gas flow. The continuity equations (equations (Nbb2)) may be written as

$$\frac{\partial}{\partial t}(\rho_N \alpha_N) + \frac{\partial(\rho_N \alpha_N u_{Ni})}{\partial x_i} = \mathcal{I}_N \qquad \text{(Nnb1)}$$

where $N = C$ and $N = D$ refer to the continuous and disperse phases respectively. We shall see that it is convenient to define a *loading* parameter, ξ, as

$$\xi = \frac{\rho_D \alpha_D}{\rho_C \alpha_C} \qquad \text{(Nnb2)}$$

and that the continuity equations have an important bearing on the variations in the value of ξ within the flow. Note that the mixture density, ρ, is then

$$\rho = \rho_C \alpha_C + \rho_D \alpha_D = (1+\xi)\rho_C \alpha_C \qquad \text{(Nnb3)}$$

The momentum and energy equations for the individual phases (equations (Nbe8) and (Nbh13)) are respectively

$$\rho_N \alpha_N \left[\frac{\partial u_{Nk}}{\partial t} + u_{Ni}\frac{\partial u_{Nk}}{\partial x_i}\right] = \alpha_N \rho_N g_k + \mathcal{F}_{Nk} - \mathcal{I}_N u_{Nk} - \delta_N \left[\frac{\partial p}{\partial x_k} - \frac{\partial \sigma^D_{Cki}}{\partial x_i}\right] \qquad \text{(Nnb4)}$$

$$\rho_N \alpha_N c_{vN}\left[\frac{\partial T_N}{\partial t} + u_{Ni}\frac{\partial T_N}{\partial x_i}\right] = \delta_N \sigma_{Cij}\frac{\partial u_{Ci}}{\partial x_j} + \mathcal{Q}_N + \mathcal{W}_N + \mathcal{Q}\mathcal{I}_N + \mathcal{F}_{Ni}(u_{Di} - u_{Ni}) - (e^*_N - u_{Ni}u_{Ni})\mathcal{I}_N \qquad \text{(Nnb5)}$$

and, when summed over all the phases, these lead to the following combined continuity, momentum and energy equations (equations (Nbb5), (Nbe9) and (Nbh14)):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}\left(\sum_N \rho_N \alpha_N u_{Ni}\right) = 0 \qquad \text{(Nnb6)}$$

$$\frac{\partial}{\partial t}\left(\sum_N \rho_N \alpha_N u_{Nk}\right) + \frac{\partial}{\partial x_i}\left(\sum_N \rho_N \alpha_N u_{Ni} u_{Nk}\right) = \rho g_k - \frac{\partial p}{\partial x_k} + \frac{\partial \sigma^D_{Cki}}{\partial x_i} \qquad \text{(Nnb7)}$$

$$\sum_N \left[\rho_N \alpha_N c_{vN}\left\{\frac{\partial T_N}{\partial t} + u_{Ni}\frac{\partial T_N}{\partial x_i}\right\}\right] = \sigma_{Cij}\frac{\partial u_{Ci}}{\partial x_j} - \mathcal{F}_{Di}(u_{Di} - u_{Ci}) - \mathcal{I}_D(e^*_D - e^*_C) + \sum_N u_{Ni}u_{Ni}\mathcal{I}_N \qquad \text{(Nnb8)}$$

To these equations of motion, we must add equations of state for both phases. Throughout this chapter it will be assumed that the continuous phase is an ideal gas and that the disperse phase is an incompressible solid. Moreover, temperature and velocity gradients in the vicinity of the interface will be neglected.