Separated Flow Limits: Kelvin-Helmholtz Instability

Separated flow regimes such as stratified horizontal flow or vertical annular flow can become unstable when waves form on the interface between the two fluid streams (subscripts 1 and 2). As indicated in figure 1, the densities of the fluids will be denoted by ρ_1 and ρ_2 and the velocities by u_1 and u_2. If these waves continue to grow in amplitude they will cause a transition to another flow regime, typically one with greater intermittency and involving plugs or slugs. Therefore, in order to determine this particular boundary of the separated flow regime, it is necessary to investigate the potential growth of the interfacial waves, whose wavelength will be denoted by λ (wavenumber, $\kappa = 2\pi/\lambda$). Studies of such waves have a long history originating with the work of Kelvin and Helmholtz and the phenomena they revealed have come to be called Kelvin-Helmholtz instabilities (see, for example, Yih 1965). In general this class of instabilities involves the interplay between at least two of the following three types of forces:

- a buoyancy force due to gravity and proportional to the difference in the densities of the two fluids. This can be characterized by $g\ell^3\Delta\rho$ where $\Delta\rho = \rho_1 - \rho_2$, g is the acceleration due to gravity and ℓ is a typical dimension of the waves. This force may be stabilizing or destabilizing depending on the orientation of gravity, g, relative to the two fluid streams. In a horizontal flow in which the upper fluid is lighter than the lower fluid the force is stabilizing. When the reverse is true the buoyancy force is destabilizing and this causes Rayleigh-Taylor instabilities. When the streams are vertical as in vertical annular flow the role played by the buoyancy force is less clear.
- a surface tension force characterized by $S\ell$ that is always stabilizing.
- a Bernoulli effect that implies a change in the pressure acting on the interface caused by a change in velocity resulting from the displacement, a of that surface. For example, if the upward displacement of the point A in figure 1 were to cause an increase in the local velocity of fluid 1 and a decrease in the local velocity of fluid 2, this would imply an induced pressure difference at the point A that would increase the amplitude of the distortion, a. Such Bernoulli forces depend on the difference in the velocity of the two streams, $\Delta u = u_1 - u_2$, and are characterized by $\rho(\Delta u)^2\ell^2$ where ρ and ℓ are a characteristic density and dimension of the flow.

The interplay between these forces is most readily illustrated by a simple example. Neglecting viscous effects, one can readily construct the planar, incompressible potential flow solution for two semi-infinite horizontal streams separated by a plane horizontal interface (as in figure 1) on which small amplitude waves have formed. Then it is readily shown (Lamb 1879, Yih 1965) that Kelvin-Helmholtz instability will occur when

$$\frac{g\Delta\rho}{\kappa} + S\kappa - \frac{\rho_1\rho_2(\Delta u)^2}{\rho_1 + \rho_2} < 0 \qquad \text{(Njo1)}$$

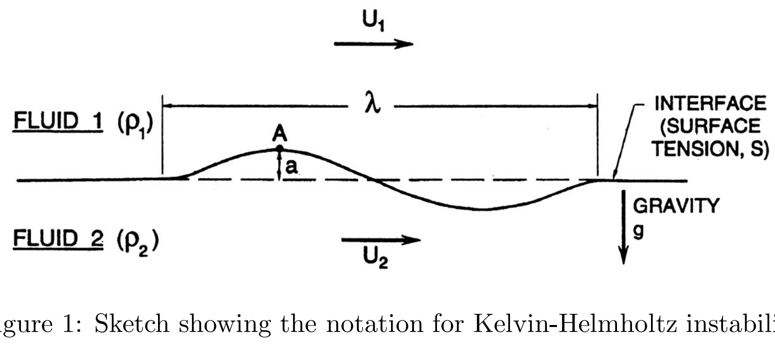

Figure 1: Sketch showing the notation for Kelvin-Helmholtz instability.

The contributions from the three previously mentioned forces are self-evident. Note that the surface tension effect is stabilizing since that term is always positive, the buoyancy effect may be stabilizing or destabilizing depending on the sign of $\Delta\rho$ and the Bernoulli effect is always destabilizing. Clearly, one subset of this class of Kelvin-Helmholtz instabilities are the Rayleigh-Taylor instabilities that occur in the absence of flow ($\Delta u = 0$) when $\Delta\rho$ is negative. In that static case, the above relation shows that the interface is unstable to all wave numbers less than the critical value, $\kappa = \kappa_c$, where

$$\kappa_c = \left(\frac{g(-\Delta\rho)}{S} \right)^{\frac{1}{2}} \tag{Njo2}$$

In the next two sections we shall focus on the instabilities induced by the destabilizing Bernoulli effect for these can often cause instability of a separated flow regime.