Frictional Loss in Separated Flow

The Lockhart-Martinelli and Martinelli- Nelson correlations attempt to predict the frictional pressure gradient in two-component or two-phase flows in pipes of constant cross-sectional area, A. It is assumed that these multiphase flows consist of two separate co-current streams that, for convenience, we will refer to as the liquid and the gas though they could be any two immiscible fluids. The correlations use the results for the frictional pressure gradient in single phase pipe flows of each of the two fluids. In two-phase flow, the volume fraction is often changing as the mixture progresses along the pipe and such phase change necessarily implies acceleration or deceleration of the fluids. Associated with this acceleration is an acceleration component of the pressure gradient that is addressed in a later section dealing with the Martinelli-Nelson correlation. Obviously, it is convenient to begin with the simpler, two-component case (the Lockhart-Martinelli correlation); this also neglects the effects of changes in the fluid densities with distance, s, along the pipe axis so that the fluid velocities also remain invariant with s. Moreover, in all cases, it is assumed that the hydrostatic pressure gradient has been accounted for so that the only remaining contribution to the pressure gradient, $-dp/ds$, is that due to the wall shear stress, τ_w. A simple balance of forces requires that

$$-\frac{dp}{ds} = \frac{P}{A}\tau_w \tag{Nkf1}$$

where P is the perimeter of the cross-section of the pipe. For a circular pipe, $P/A = 4/d$, where d is the pipe diameter and, for non-circular cross-sections, it is convenient to define a *hydraulic diameter*, $4A/P$. Then, defining the dimensionless friction coefficient, C_f, as

$$C_f = \tau_w / \frac{1}{2}\rho j^2 \tag{Nkf2}$$

the more general form of equation (Nkb1) becomes

$$-\frac{dp}{ds} = 2C_f \rho j^2 \frac{P}{4A} \tag{Nkf3}$$

In single phase flow the coefficient, C_f, is a function of the Reynolds number, $\rho dj/\mu$, of the form

$$C_f = \mathcal{K}\left\{\frac{\rho dj}{\mu}\right\}^{-m} \tag{Nkf4}$$

where $\mathcal{K}$ is a constant that depends on the roughness of the pipe surface and will be different for laminar and turbulent flow. The index, m, is also different, being 1 in the case of laminar flow and $\frac{1}{4}$ in the case of turbulent flow.

These relations from single phase flow are applied to the two cocurrent streams in the following way. First, we define hydraulic diameters, d_L and d_G, for each of the two streams and define corresponding area ratios, κ_L and κ_G, as

$$\kappa_L = 4A_L/\pi d_L^2 \quad ; \quad \kappa_G = 4A_G/\pi d_G^2 \tag{Nkf5}$$

where $A_L = A(1-\alpha)$ and $A_G = A\alpha$ are the actual cross-sectional areas of the two streams. The quantities κ_L and κ_G are shape parameters that depend on the geometry of the flow pattern. In the absence of any specific information on this geometry, one might choose the values pertinent to streams of circular cross-section, namely $\kappa_L = \kappa_G = 1$, and the commonly used form of the Lockhart-Martinelli correlation employs these values. However, as an alternative example, we shall also present data for the case of annular flow in which the liquid coats the pipe wall with a film of uniform thickness and the gas flows in a cylindrical core. When the film is thin, it follows from the annular flow geometry that

$$\kappa_L = 1/(1-\alpha) \quad ; \quad \kappa_G = 1 \tag{Nkf6}$$

where it has been assumed that only the exterior perimeter of the annular liquid stream experiences significant shear stress.

In summary, the basic geometric relations yield

$$\alpha = 1 - \kappa_L d_L^2/d^2 = \kappa_G d_G^2/d^2 \tag{Nkf7}$$

Then, the pressure gradient in each stream is assumed given by the following coefficients taken from single phase pipe flow:

$$C_{fL} = \mathcal{K}_L \left\{ \frac{\rho_L d_L u_L}{\mu_L} \right\}^{-m_L} \quad ; \quad C_{fG} = \mathcal{K}_G \left\{ \frac{\rho_G d_G u_G}{\mu_G} \right\}^{-m_G} \tag{Nkf8}$$

and, since the pressure gradients must be the same in the two streams, this imposes the following relation between the flows:

$$-\frac{dp}{ds} = \frac{2\rho_L u_L^2 \mathcal{K}_L}{d_L} \left\{ \frac{\rho_L d_L u_L}{\mu_L} \right\}^{-m_L} = \frac{2\rho_G u_G^2 \mathcal{K}_G}{d_G} \left\{ \frac{\rho_G d_G u_G}{\mu_G} \right\}^{-m_G} \tag{Nkf9}$$

In the above, m_L and m_G are 1 or $\frac{1}{4}$ depending on whether the stream is laminar or turbulent. It follows that there are four permutations namely:

- both streams are laminar so that $m_L = m_G = 1$, a permutation denoted by the double subscript LL
- a laminar liquid stream and a turbulent gas stream so that $m_L = 1$, $m_G = \frac{1}{4}$ (LT)
- a turbulent liquid stream and a laminar gas stream so that $m_L = \frac{1}{4}$, $m_G = 1$ (TL) and
- both streams are turbulent so that $m_L = m_G = \frac{1}{4}$ (TT)

Equations (Nkf7) and (Nkf9) are the basic relations used to construct the Lockhart-Martinelli correlation. However, the solutions to these equations are normally and most conveniently presented in non-dimensional form by defining the following dimensionless pressure gradient parameters:

$$\phi_L^2 = \frac{\left(\frac{dp}{ds}\right)_{actual}}{\left(\frac{dp}{ds}\right)_L} \quad ; \quad \phi_G^2 = \frac{\left(\frac{dp}{ds}\right)_{actual}}{\left(\frac{dp}{ds}\right)_G} \tag{Nkf10}$$

where $(dp/ds)_L$ and $(dp/ds)_G$ are respectively the hypothetical pressure gradients that would occur in the same pipe if only the liquid flow were present and if only the gas flow were present. The ratio of these two hypothetical gradients, Ma^2, given by

$$Ma^2 = \frac{\phi_G^2}{\phi_L^2} = \frac{\left(\frac{dp}{ds}\right)_L}{\left(\frac{dp}{ds}\right)_G} = \frac{\rho_L}{\rho_G} \frac{G_G^2}{G_L^2} \frac{\mathcal{K}_G}{\mathcal{K}_L} \frac{\left\{\frac{G_G d}{\mu_G}\right\}^{-m_G}}{\left\{\frac{G_L d}{\mu_L}\right\}^{-m_L}} \tag{Nkf11}$$

defines the Martinelli parameter, Ma, and allows presentation of the solutions to equations (Nkf7) and (Nkf9) in a convenient parametric form. Using the definitions of equations (Nkf10), the non-dimensional forms of equations (Nkf7) become

$$\alpha = 1 - \kappa_L^{-(1+m_L)/(m_L-5)} \phi_L^{4/(m_L-5)} = \kappa_G^{-(1+m_G)/(m_G-5)} \phi_G^{4/(m_G-5)} \tag{Nkf12}$$

and the solution of these equations produces the Lockhart-Martinelli prediction of the non-dimensional pressure gradient.

To summarize: for given values of

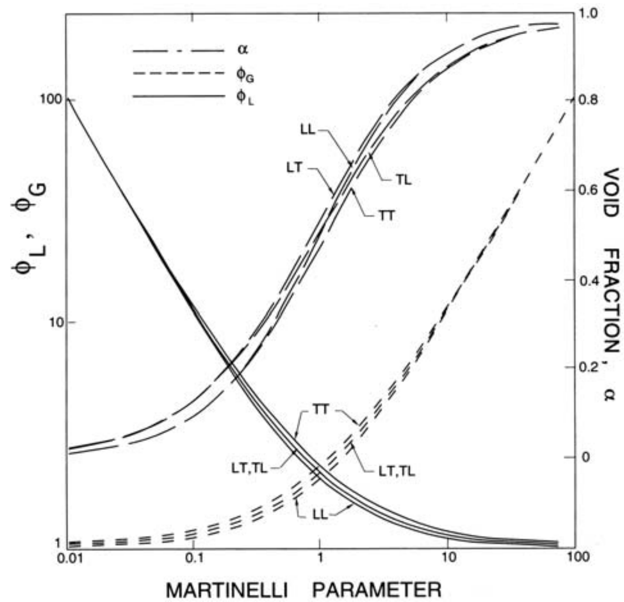

Figure 1: The Lockhart-Martinelli correlation results for ϕ_L and ϕ_G and the void fraction, α, as functions of the Martinelli parameter, Ma, for the case, $\kappa_L = \kappa_G = 1$. Results are shown for the four laminar and turbulent stream permutations, LL, LT, TL and TT.

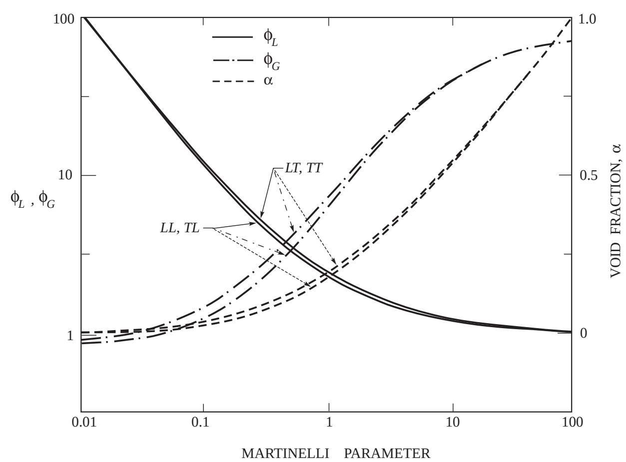

Figure 2: As figure 1 but for the annular flow case with $\kappa_L = 1/(1-\alpha)$ and $\kappa_G = 1$.

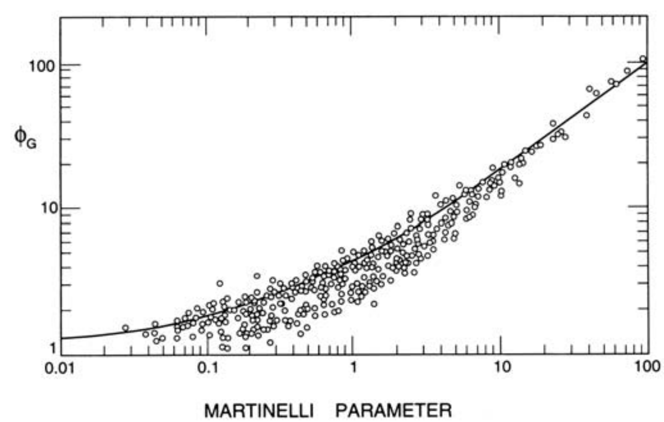

Figure 3: Comparison of the Lockhart-Martinelli correlation (the TT case) for ϕ_G (solid line) with experimental data. Adapted from Turner and Wallis (1965).

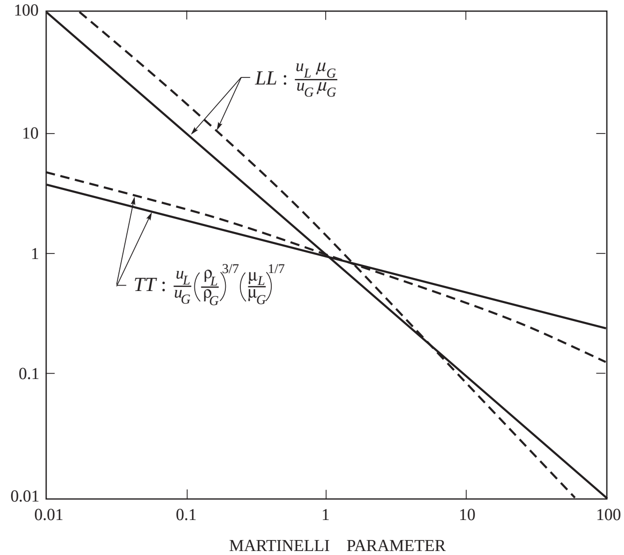

Figure 4: Ratios demonstrating the velocity ratio, u_L/u_G, implicit in the Lockhart-Martinelli correlation as functions of the Martinelli parameter, Ma, for the LL and TT cases. Solid lines: $\kappa_L = \kappa_G = 1$; dashed lines: $\kappa_L = 1/(1-\alpha)$, $\kappa_G = 1$.

- the fluid properties, ρ_L, ρ_G, μ_L and μ_G
- a given type of flow LL, LT, TL or TT along with the single phase correlation constants, m_L, m_G, $\mathcal{K}_L$ and $\mathcal{K}_G$
- given values or expressions for the parameters of the flow pattern geometry, κ_L and κ_G
- and a given value of α

equations (Nkf12) can be solved to find the non-dimensional solution to the flow, namely the values of ϕ_L^2 and ϕ_G^2. The value of Ma^2 also follows and the rightmost expression in equation (Nkf11) then yields a relation between the liquid mass flux, G_L, and the gas mass flux, G_G. Thus, if one is also given just **one** mass flux (often this will be the total mass flux, G), the solution will yield the individual mass fluxes, the mass quality and other flow properties. Alternatively one could begin the calculation with the mass quality rather than the void fraction and find the void fraction as one of the results. Finally the pressure gradient, dp/ds, follows from the values of ϕ_L^2 and ϕ_G^2.

The solutions for the cases $\kappa_L = \kappa_G = 1$ and $\kappa_L = 1/2(1-\alpha)$, $\kappa_G = 1$ are presented in figures 1 and 2 and the comparison of these two figures yields some measure of the sensitivity of the results to the flow geometry parameters, κ_L and κ_G. Similar charts are commonly used in the manner described above to obtain solutions for two-component gas/liquid flows in pipes. A typical comparison of the Lockhart-Martinelli prediction with the experimental data is presented in figure 3. Note that the scatter in the data is significant (about a factor of 3 in ϕ_G) and that the Lockhart-Martinelli prediction often yields an overestimate of the friction or pressure gradient. This is the result of the assumption that the entire perimeter of both phases experiences static wall friction. This is not the case and part of the perimeter of each phase is in contact with the other phase. If the interface is smooth this could result in a decrease in the friction; one the other hand a roughened interface could also result in increased interfacial friction.

It is important to recognize that there are many deficiencies in the Lockhart-Martinelli approach. First, it is assumed that the flow pattern consists of two parallel streams and any departure from this topology could result in substantial errors. In figure 4, the ratios of the velocities in the two streams which are implicit in the correlation (and follow from equation (Nkf11)) are plotted against the Martinelli parameter. Note that large velocity differences appear to be predicted at void fractions close to unity. Since the flow is likely to transition to mist flow in this limit and since the relative velocities in the mist flow are unlikely to become large, it seems inevitable that the correlation would become quite inaccurate at these high void fractions. Similar inaccuracies seem inevitable at low void fraction. Indeed, it appears that the Lockhart-Martinelli correlations work best under conditions that do not imply large velocity differences. Figure 4 demonstrates that smaller velocity differences are expected for turbulent flow (TT) and this is mirrored in better correlation with the experimental results in the turbulent flow case (Turner and Wallis 1965).

Second, there is the previously discussed deficiency regarding the suitability of assuming that the perimeters of both phases experience friction that is effectively equivalent to that of a static solid wall. A third source of error arises because the multiphase flows are often unsteady and this yields a multitude of quadratic interaction terms that contribute to the mean flow in the same way that Reynolds stress terms contribute to turbulent single phase flow.