Homogeneous Flow Friction

When the multiphase flow or slurry is thoroughly mixed the pressure drop can be approximated by the friction coefficient for a single-phase flow with the mixture density, ρ (equation (Nac8)) and the same total volumetric flux, $j = j_S + j_L$, as the multiphase flow. We exemplify this using the slurry pipeline data from the preceding section assuming that $\alpha = \beta$ (which does tend to be the case in horizontal homogeneous flows) and setting $j = j_L/(1-\alpha)$. Then the ratio of the base friction coefficient at finite loading, $C_f(\alpha)$, to the friction coefficient for the continuous phase alone, $C_f(0)$, should be given by

$$\frac{C_f(\alpha)}{C_f(0)} = \frac{(1+\alpha\rho_S/\rho_L)}{(1-\alpha)^2} \tag{Nkc1}$$

A comparison between this expression and the data from the base curves of Lazarus and Neilsen is included in figure 1 and demonstrates a reasonable agreement.

Thus a flow regime that is homogeneous or thoroughly mixed can usually be modeled as a single phase flow with an effective density, volume flow rate and viscosity. In these circumstances the orientation of the pipe appears to make little difference. Often these correlations also require an effective mixture viscosity. In the above example, an effective kinematic viscosity of the multiphase flow could have been incorporated in the expression (Nkc1); however, this has little effect on the comparison in figure 1 especially under the turbulent conditions in which most slurry pipelines operate.

Wallis (1969) includes a discussion of homogeneous flow friction correlations for both laminar and turbulent flow. In laminar flow, most correlations use the mixture density as the effective density and the total volumetric flux, j, as the velocity as we did in the above example. A wide variety of mostly empirical expressions are used for the effective viscosity, μ_e. In low volume fraction suspensions of solid particles, Einstein's (1906) classical effective viscosity given by

$$\mu_e = \mu_C(1+5\alpha/2) \tag{Nkc2}$$

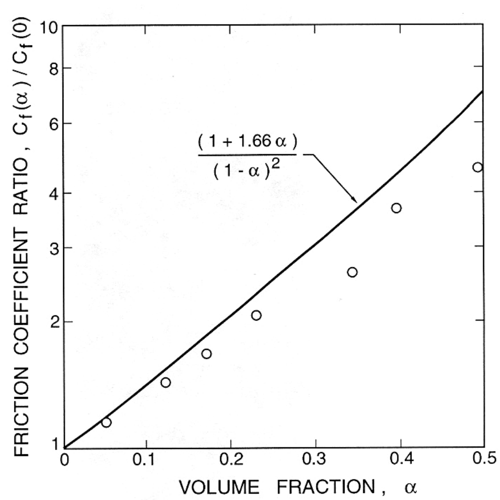

Figure 1: The ratio of the *base curve* friction coefficient at finite loading, $C_f(\alpha)$, to the friction coefficient for the continuous phase alone, $C_f(0)$. Equation **??** (line) is compared with the data of Lazarus and Neilsen (1978).

is appropriate though this expression loses validity for volume fractions greater than a few percent. In emulsions with droplets of viscosity, μ_D, the extension of Einstein's formula,

$$\mu_e = \mu_C \left\{ 1 + \frac{5\alpha}{2} \frac{(\mu_D + 2\mu_C/5)}{(\mu_D + \mu_C)} \right\} \quad \text{(Nkc3)}$$

is the corresponding expression (Happel and Brenner 1965). More empirical expressions for μ_e are typically used at higher volume fractions.

As discussed in section (Nca), turbulence in multiphase flows introduces another set of complicated issues. Nevertheless as was demonstrated by the above example, the effective single phase approach to pipe friction seems to produce moderately accurate results in homogeneous flows. The comparison in figure 2 shows that the errors in such an approach are about ±25%. The presence of particles, particularly solid particles, can act like surface roughness, enhancing turbulence in many applications. Consequently, turbulent friction factors for homogeneous flow tend to be similar to the values obtained for single phase flow in rough pipes, values around 0.005 being commonly experienced (Wallis 1969).

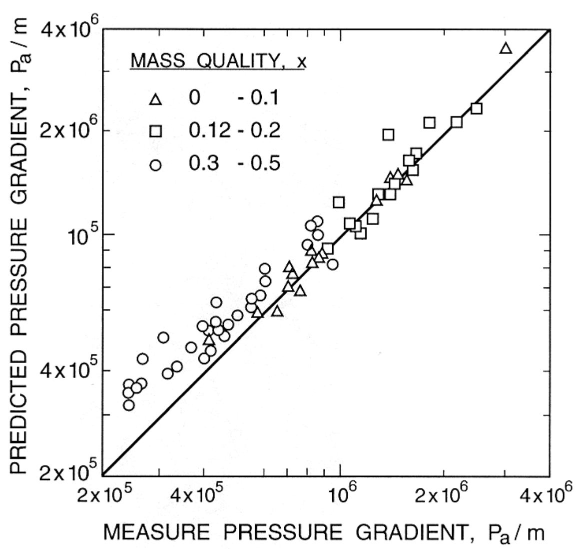

Figure 2: Comparison of the measured friction coefficient with that using the homogeneous prediction for steam/water flows of various mass qualities in a 0.3cm diameter tube. From Owens (1961).