Growing or Collapsing Bubbles

When the volume of a bubble changes significantly, that growth or collapse can also have a substantial effect upon its translation. In this section we return to the discussion of high Re flow in section (Neg) and specifically address the effects due to bubble growth or collapse. A bubble that grows or collapses close to a boundary may undergo translation due to the asymmetry induced by that boundary. A relatively simple example of the analysis of this class of flows is the case of the growth or collapse of a spherical bubble near a plane boundary, a problem first solved by Herring (1941) (see also Davies and Taylor 1942, 1943). Assuming that the only translational motion of the bubble is perpendicular to the plane boundary with velocity, W, the geometry of the bubble and its image in the boundary will be as shown in figure 1. For convenience, we define additional polar coordinates, $(\breve{r}, \breve{\theta})$, with origin at the center of the image bubble. Assuming inviscid, irrotational flow, Herring (1941) and Davies and Taylor (1943) constructed the velocity potential, ϕ, near the bubble by considering an expansion in terms of R/H where H is the distance of the bubble center from the boundary. Neglecting all terms that are of order R^3/H^3 or higher, the velocity potential can be obtained by superimposing the individual contributions from the bubble source/sink, the image source/sink, the bubble translation dipole, the image dipole, and one correction factor described below. This combination yields

$$\phi = -\frac{R^2\dot{R}}{r} - \frac{WR^3\cos\theta}{2r^2} \pm \left\{ -\frac{R^2\dot{R}}{\breve{r}} + \frac{WR^3\cos\breve{\theta}}{2\breve{r}^2} - \frac{R^5\dot{R}\cos\theta}{8H^2r^2} \right\} \tag{Nff1}$$

The first and third terms are the source/sink contributions from the bubble and the image respectively. The second and fourth terms are the dipole contributions due to the translation of the bubble and the image. The last term arises because the source/sink in the bubble needs to be displaced from the bubble center by an amount $R^3/8H^2$ normal to the wall in order to satisfy the boundary condition on the surface of the bubble to order R^2/H^2. All other terms of order R^3/H^3 or higher are neglected in this analysis assuming that the bubble is sufficiently far from the boundary so that $H \gg R$. Finally, the sign choice on the last three terms of equation (Nff1) is as follows: the upper, positive sign pertains to the case of a solid boundary and the lower, negative sign provides an approximate solution for a free surface boundary.

It remains to use this solution to determine the translational motion, $W(t)$, normal to the boundary. This is accomplished by invoking the condition that there is no net force on the bubble. Using the unsteady

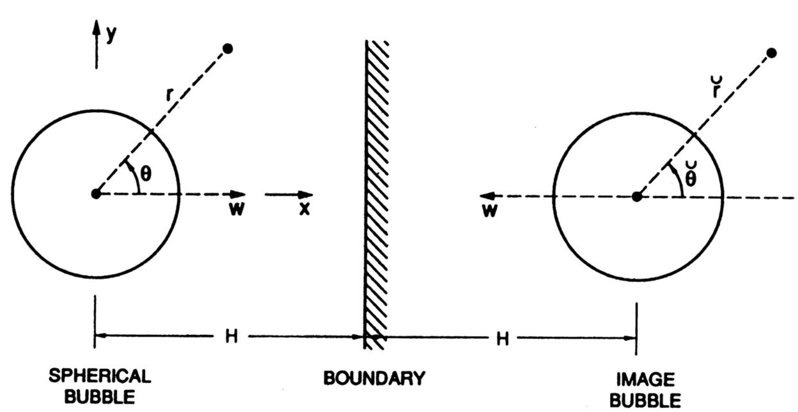

Figure 1: Schematic of a bubble undergoing growth or collapse close to a plane boundary. The associated translational velocity is denoted by W.

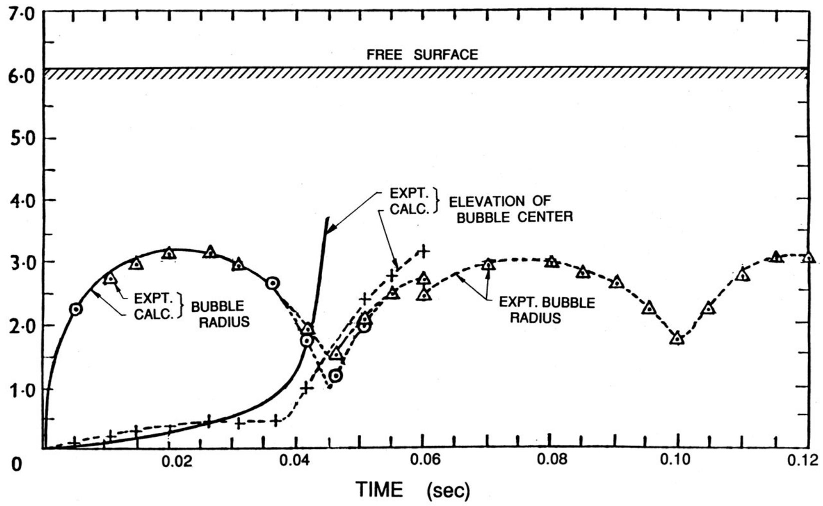

Figure 2: Data from Davies and Taylor (1943) on the mean radius and central elevation of a bubble in oil generated by a spark-initiated explosion of 1.32×10^6 $ergs$ situated 6.05 cm below the free surface. The two measures of the bubble radius are one half of the horizontal span (△) and one quarter of the sum of the horizontal and vertical spans (⊙). Theoretical calculations using equation (Nff3) are indicated by the solid lines.

Bernoulli equation and the velocity potential and fluid velocities obtained from equation (Nff1), Davies and Taylor (1943) evaluate the pressure at the bubble surface and thereby obtain an expression for the force, F_x, on the bubble in the x direction:

$$F_x = -\frac{2\pi}{3}\left\{\frac{d}{dt}\left(R^3W\right) \pm \frac{3}{4}\frac{R^2}{H^2}\frac{d}{dt}\left(R^3\frac{dR}{dt}\right)\right\} \tag{Nff2}$$

Adding the effect of buoyancy due to a component, g_x, of the gravitational acceleration in the x direction, Davies and Taylor then set the total force equal to zero and obtain the following equation of motion for $W(t)$:

$$\frac{d}{dt}\left(R^3W\right) \pm \frac{3}{4}\frac{R^2}{H^2}\frac{d}{dt}\left(R^3\frac{dR}{dt}\right) + \frac{4\pi R^3 g_x}{3} = 0 \tag{Nff3}$$

In the absence of gravity this corresponds to the equation of motion first obtained by Herring (1941). Many of the studies of growing and collapsing bubbles near boundaries have been carried out in the context of underwater explosions (see Cole 1948). An example illustrating the solution of equation (Nff3) and the comparison with experimental data is included in figure 2 taken from Davies and Taylor (1943).

Another application of this analysis is to the translation of cavitation bubbles near walls. Here the motivation is to understand the development of impulsive loads on the solid surface. Therefore the primary focus is on bubbles close to the wall and the solution described above is of limited value since it requires $H \gg R$. However, considerable progress has been made in recent years in developing analytical methods for the solution of the inviscid free surface flows of bubbles near boundaries (Blake and Gibson 1987). One of the concepts that is particularly useful in determining the direction of bubble translation is based on a property of the flow first introduced by Kelvin (see Lamb 1932) and called the Kelvin impulse. This vector property applies to the flow generated by a finite particle or bubble in a fluid; it is denoted by I_{Ki} and defined by

$$I_{Ki} = \rho_L \int_{S_B} \phi n_i dS \tag{Nff4}$$

where ϕ is the velocity potential of the irrotational flow, S_B is the surface of the bubble, and n_i is the outward normal at that surface (defined as positive into the bubble). If one visualizes a bubble in a fluid at rest, then the Kelvin impulse is the impulse that would have to be applied to the bubble in order to generate the motions of the fluid related to the bubble motion. Benjamin and Ellis (1966) were the first to demonstrate the value of this property in determining the interaction between a growing or collapsing bubble and a nearby boundary (see also Blake and Gibson 1987).