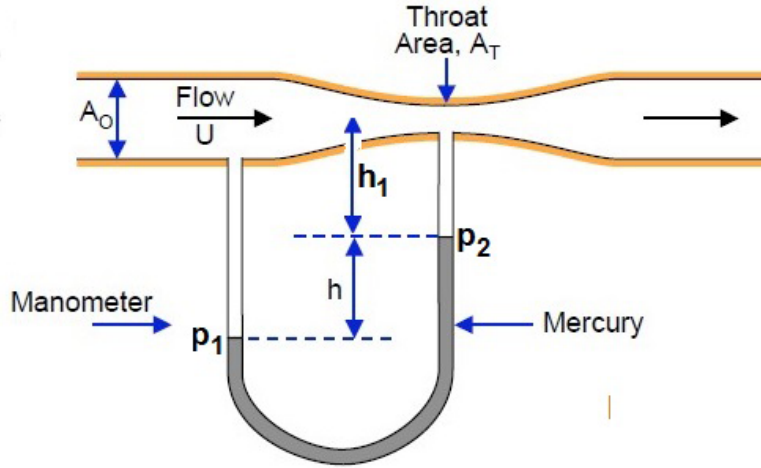


Solution to Problem 220K:

The following device, known as a Venturi meter, is used to measure the flow rate of water in a pipe of cross-sectional area A_0 . A convergent/divergent nozzle with a throat area A_T is installed in the pipe: and



pressure taps are located upstream of the nozzle and at the throat; these are connected to a water/mercury manometer as shown. When the water is flowing through the device, the manometer levels differ by an elevation, h .

Continuity requires that

$$U A_0 = U_T A_T \quad \text{so that} \quad U_T = U \frac{A_0}{A_T} \quad (1)$$

and Bernoulli's equation requires that

$$p_T - p_0 = \frac{\rho_W}{2} U^2 \left[1 - \frac{A_0^2}{A_T^2} \right] \quad (2)$$

where p_T and p_0 are the pressures in the throat and far upstream. Therefore

$$U = \left[\frac{2A_T^2(p_0 - p_T)}{\rho_W(A_0^2 - A_T^2)} \right]^{1/2} \quad (3)$$

The manometer will read

$$p_0 - p_T = (\rho_M - \rho_W)gh \quad (4)$$

and therefore the flow rate, Q , can be determined using

$$Q = U A_0 = \left[\frac{2ghA_T^2 A_0^2 (\rho_M - \rho_W)}{\rho_W(A_0^2 - A_T^2)} \right]^{1/2} \quad (5)$$