Solution to Problem 150D

Each of the two fluids experience a Couette flow and therefore the velocities in the two streams must be given by

$$u_A(y) = Cy + D$$

and

$$u_B(y) = Ey + F$$

where C, D, E and F are constants to be determined by the boundary conditions as follows:

- The no-slip boundary condition at the lower wall requires that

$$u_B(0) = 0$$

- The no-slip boundary condition at the upper wall requires that

$$u_A(H) = U$$

- The no-slip boundary condition at the interface requires that

$$u_A(H/2) = u_B(H/2)$$

- The shear stress at the interface must be the same in the two fluids so that

$$\sigma_A(H/2) = \mu_A \left(\frac{du_A}{dy}\right)_{y=H/2} = \sigma_B(H/2) = \mu_B \left(\frac{du_B}{dy}\right)_{y=H/2}$$

Utilizing these four boundary conditions allows evaluation of C, D, E and F and from these we arrive at the velocity, u^*, of the interface:

$$u^* = \frac{U}{1 + \frac{\mu_B}{\mu_A}}$$

For the given ratio of viscosities, μ_B/μ_A, this gives:

$$u^* = \frac{U}{5}$$

The apparent viscosity μ^* can be evaluated by assuming an observer who only sees the boundary walls and an apparent viscosity would conclude that this was

$$\mu^* = \sigma_A / \frac{U}{H} = \frac{8}{5}\mu_A = \frac{8}{5}\mu$$